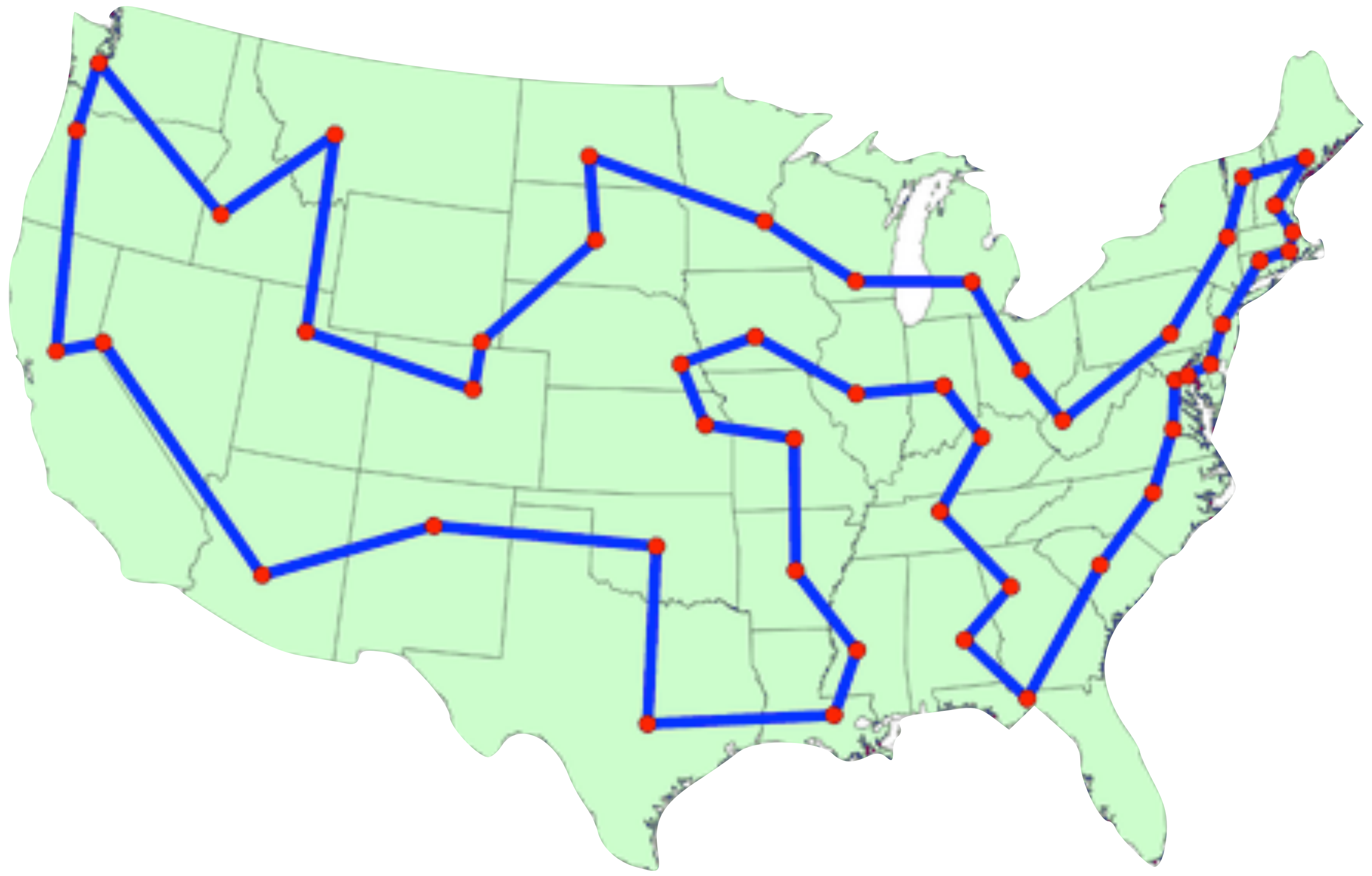
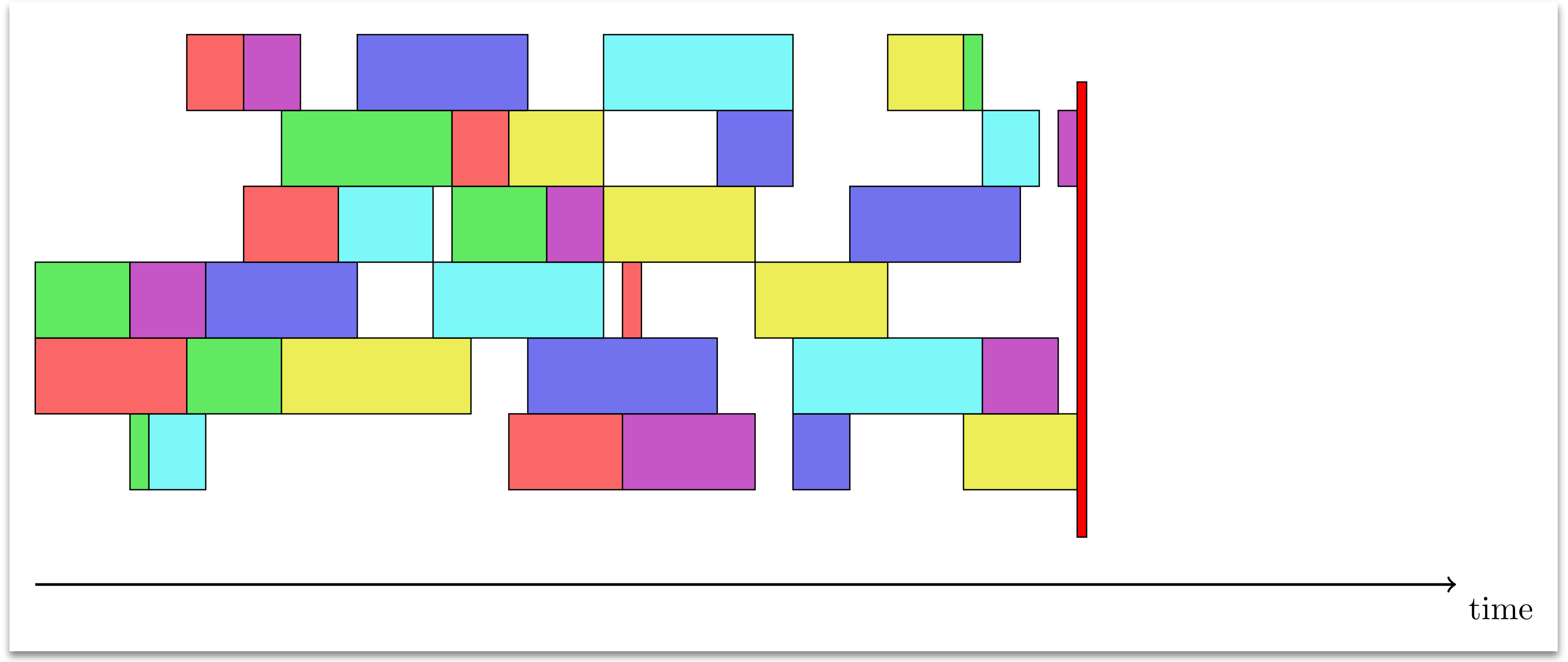
Introduction to Constraint Programming

Discrete Optimization is everywhere!

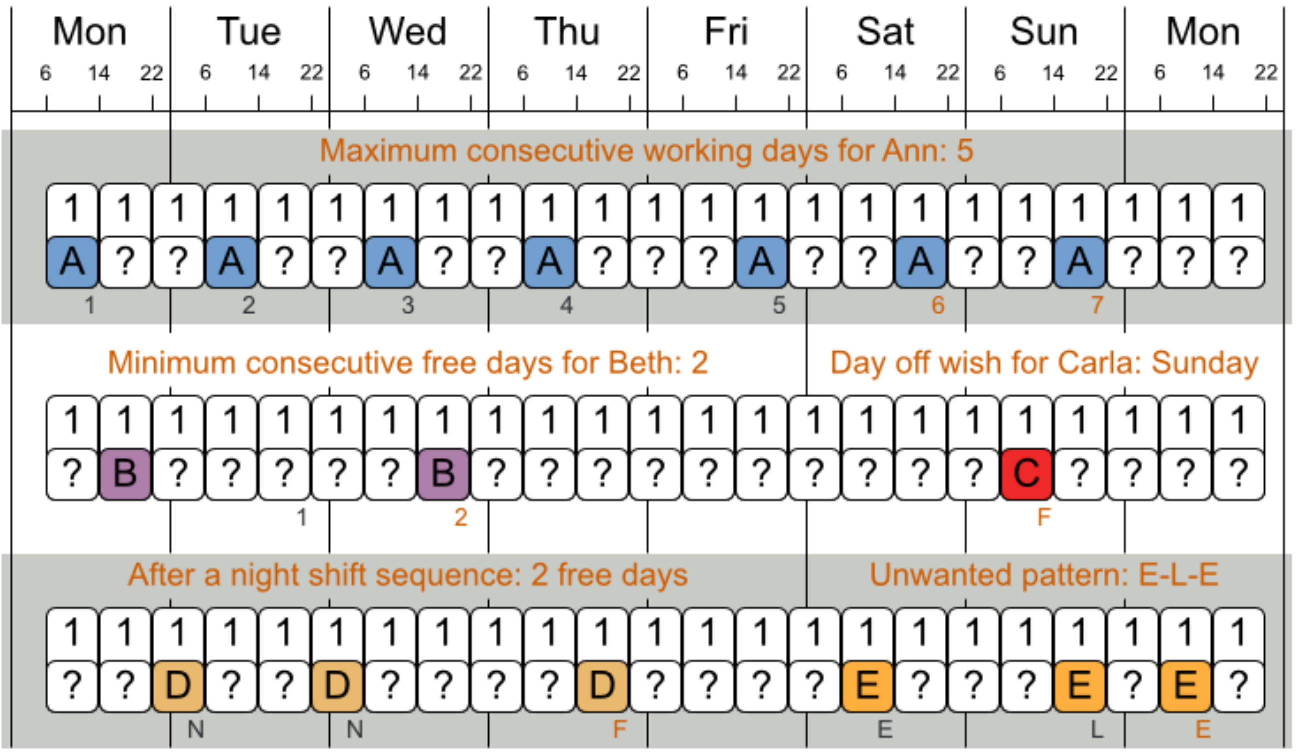
Routing



Scheduling

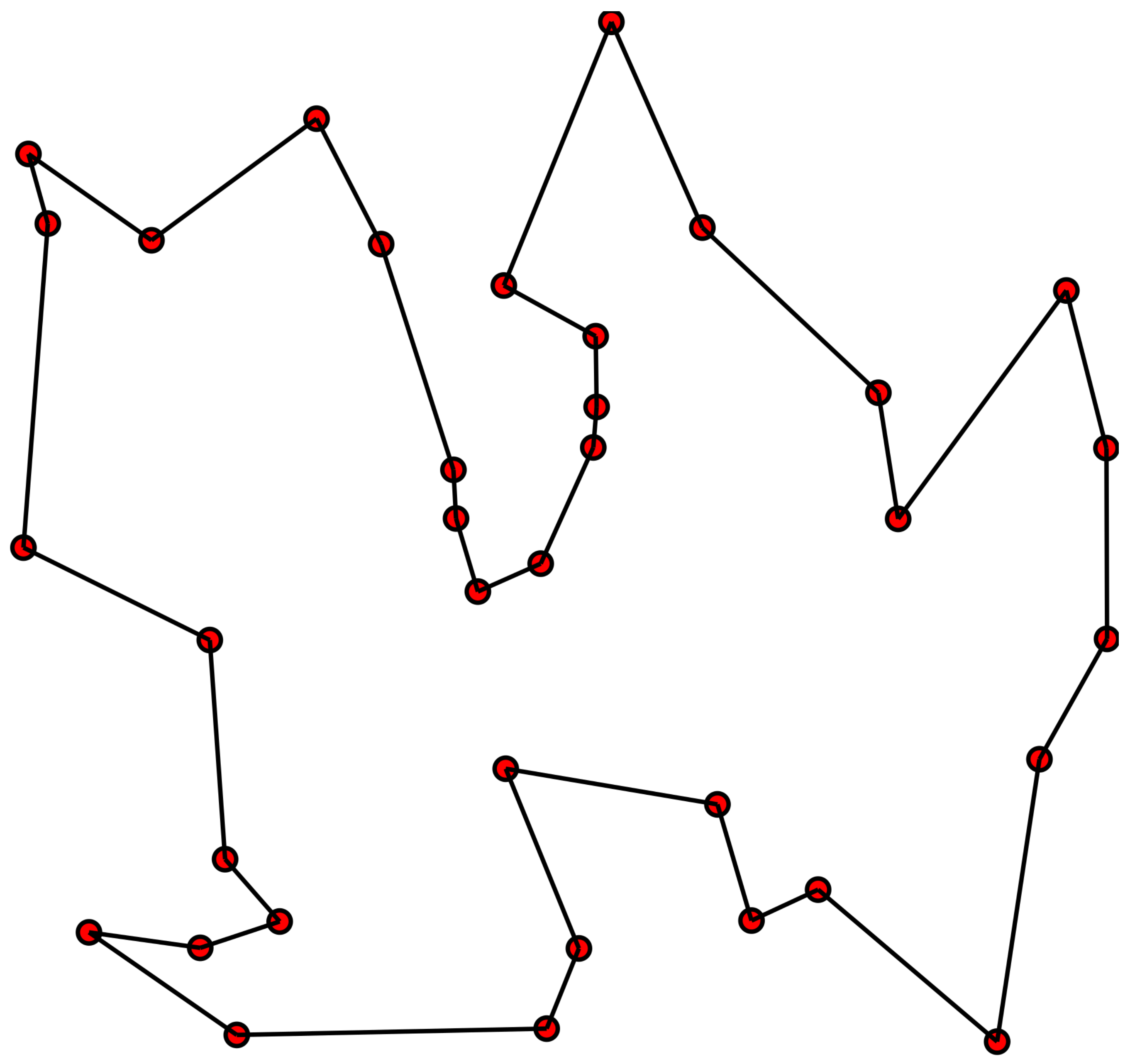


Rostering



Discrete Optimization problems are messy

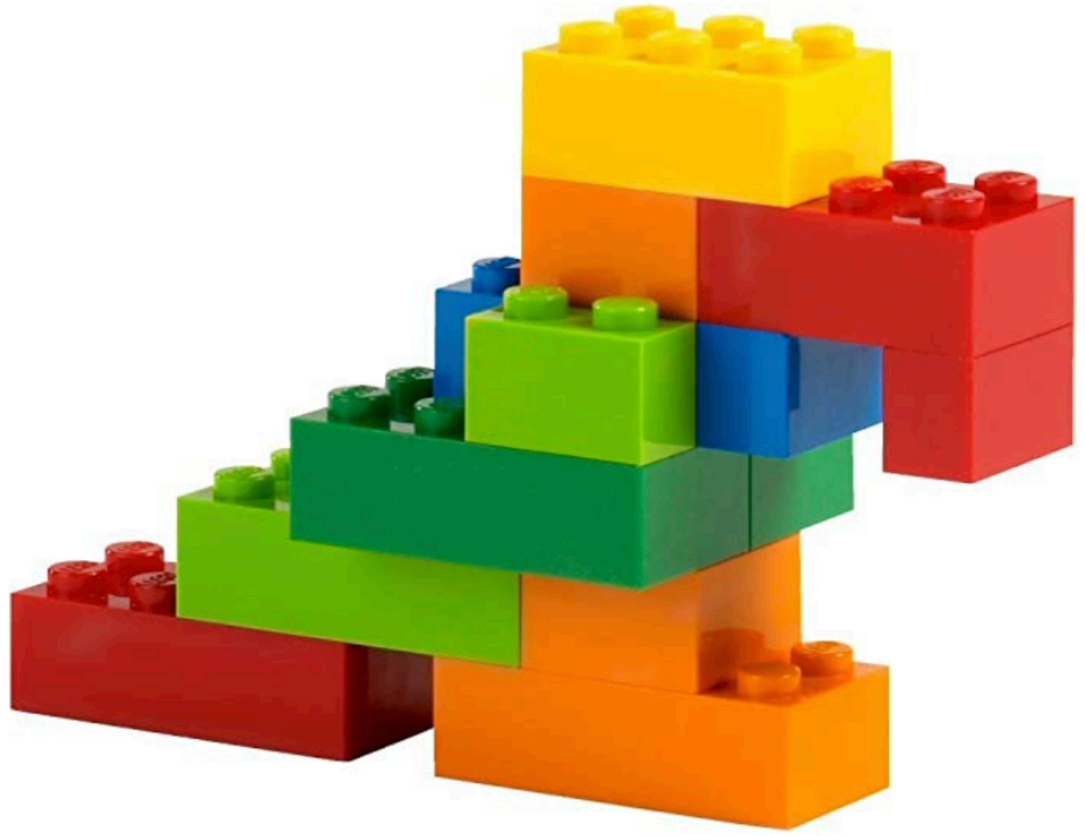
- Pure TSP only exist in text-books and student projects



- In practice you will have more than one vehicle, and dozens of constraints and strange objective functions 🤪

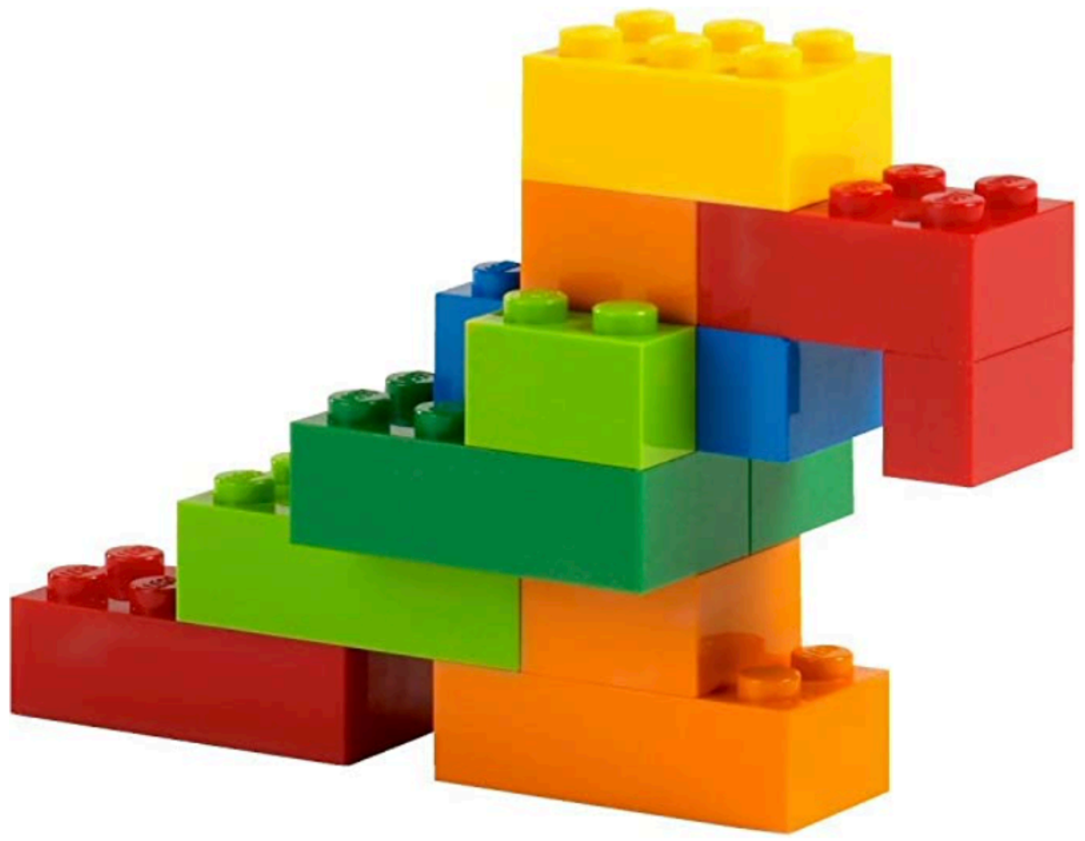
Constraint Programming

- Is a very good tool to solve messy discrete optimization problems



Constraint Programming

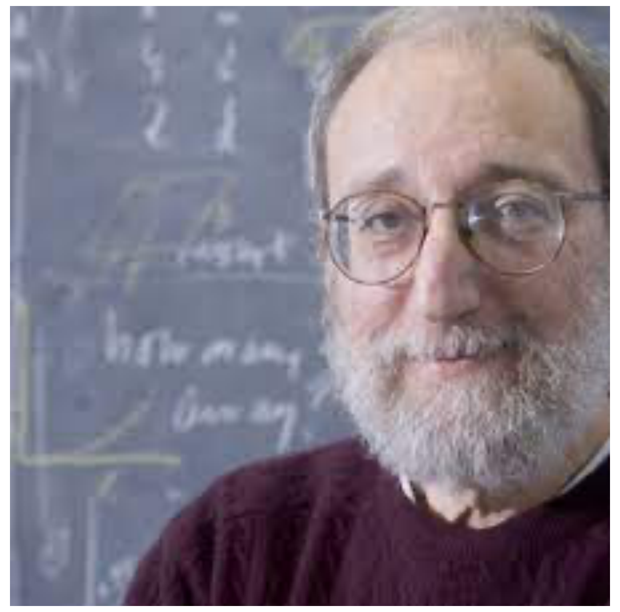
- Is a very good tool to solve messy discrete optimization problems



Constraint Programming (CP)

"Constraint programming represents one of the closest approaches computer science has yet made to the Holy Grail of programming: the user states the problem, the computer solves it." (E. Freuder)

States, you mean like this?



Not yet … rather like this:

```
range R = 1..8;
var{int} q[R] in R;
solve {
   forall(i in R, j in R: i < j) {
      q[i] ≠  q[j];
      q[i] ≠  q[j] + (j - i);
      q[i] ≠  q[j] - (j - i);
   }
}
```

but who knows in the future ;-)

State Problem = Declarative Programming

Declarative programming is a *programming paradigm* that expresses the logic of a computation without describing its control flow.

Declarative programming for solving constrained combinatorial (optimization) problems means that you express the properties of solutions that must be found by “the solver”.

CP Slogan

CP = Model (+ Search)

Model description:

user API for
declarative programming

The algorithmic part:

finding a solution that
satisfies all the constraints, etc,

usually by exploring a search tree

What will you learn ?

1) How to build this

2) How to use this

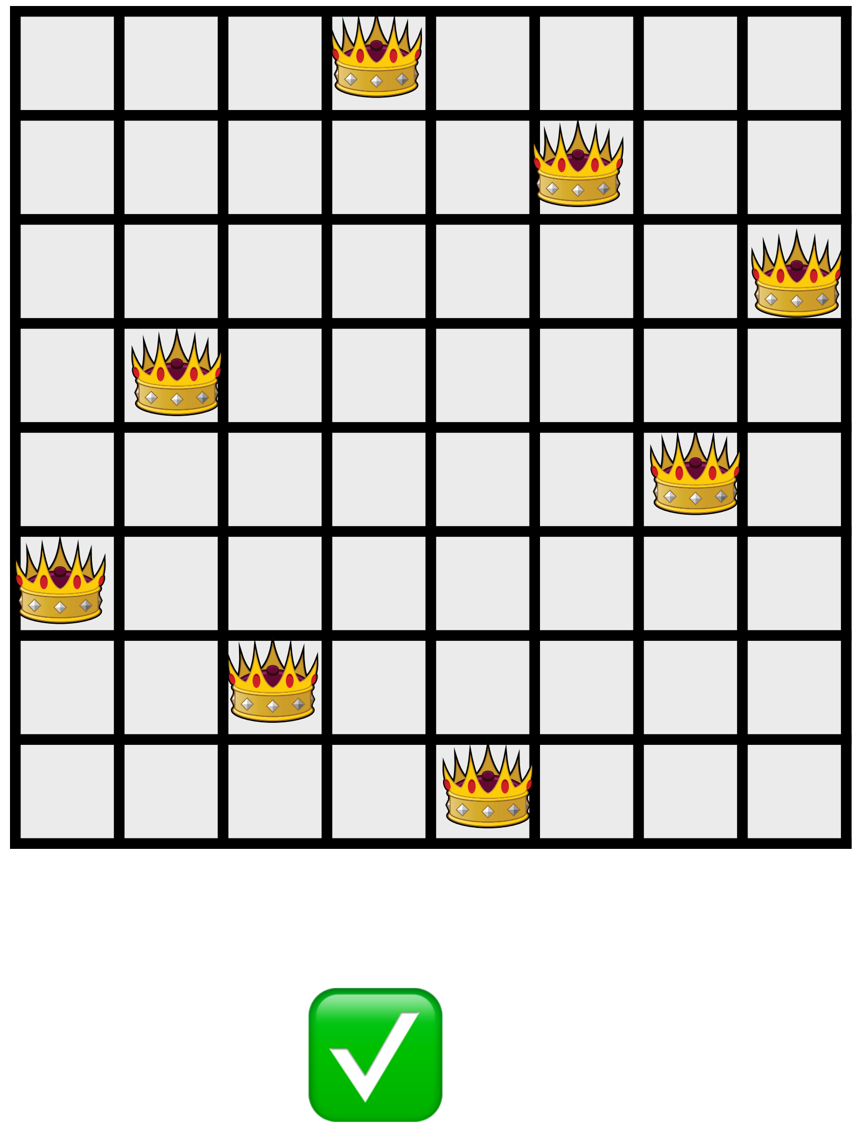
Outline

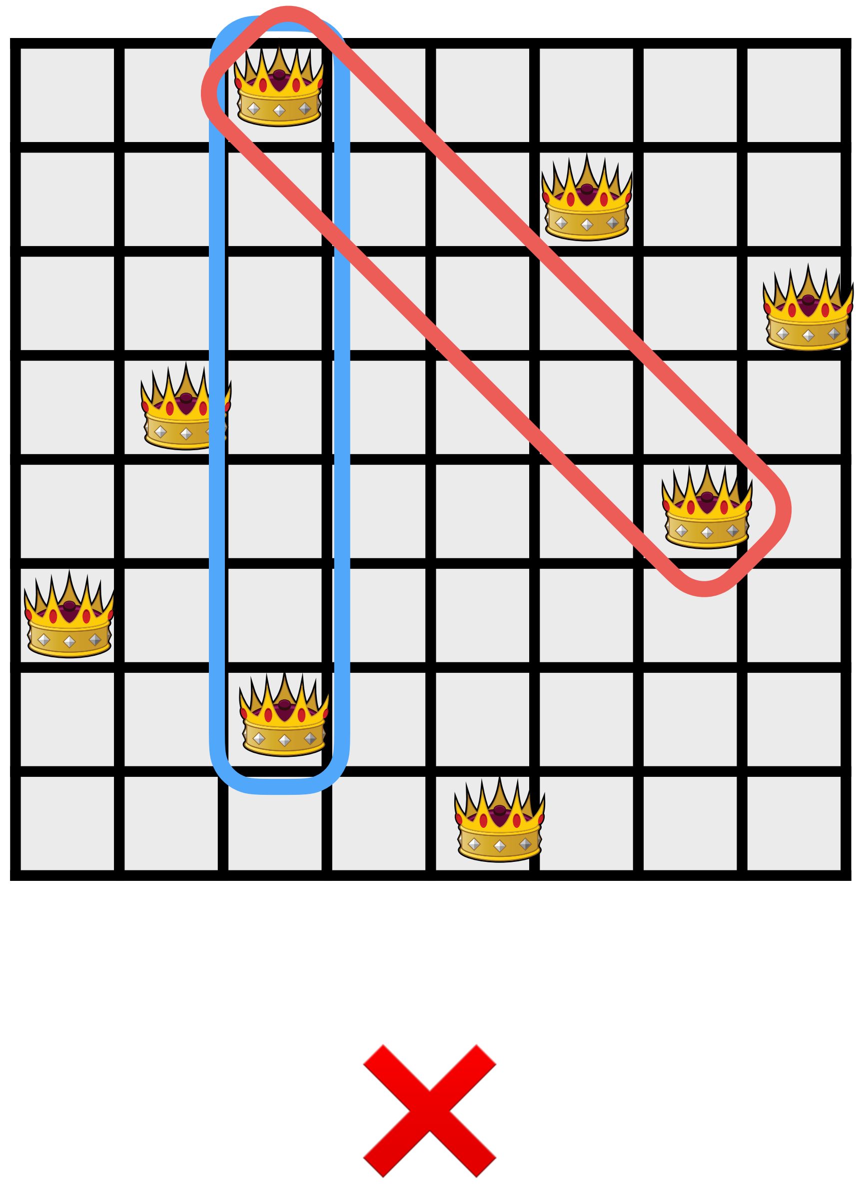
- The N-Queens Problem
- Three approaches
  - DFS + Filter
  - DFS + Prune
  - (Tiny)-CSP: make it generic and reusable:
    - Variables, domains, constraints and DFS
  - Declarative Paradigm
  - Assignment: Sudoku +
  - What's next

DFS + Filter

N-Queens Problem

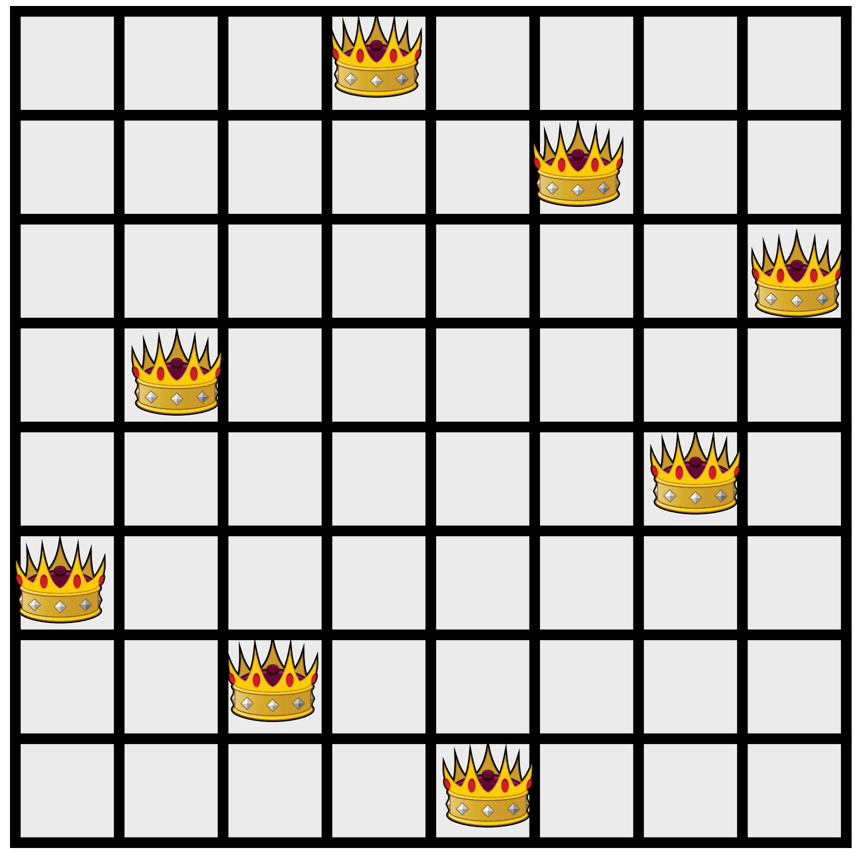
- Place eight queens on an n×n chessboard so that no two queens threaten each other;
- Thus, a solution requires that no two queens share the same row, column, or diagonal.





N-Queens: modeling considerations

A boolean {True/False} for each cell telling whether or not a queen is present



F	F	F	T	F	F	F	F
F	F	F	F	F	T	F	F
F	F	F	F	F	F	F	T
F	T	F	F	F	F	F	F
F	F	F	F	F	F	T	F
T	F	F	F	F	F	F	F
F	F	T	F	F	F	F	F
F	F	F	F	T	F	F	F

N-Queens: modeling considerations

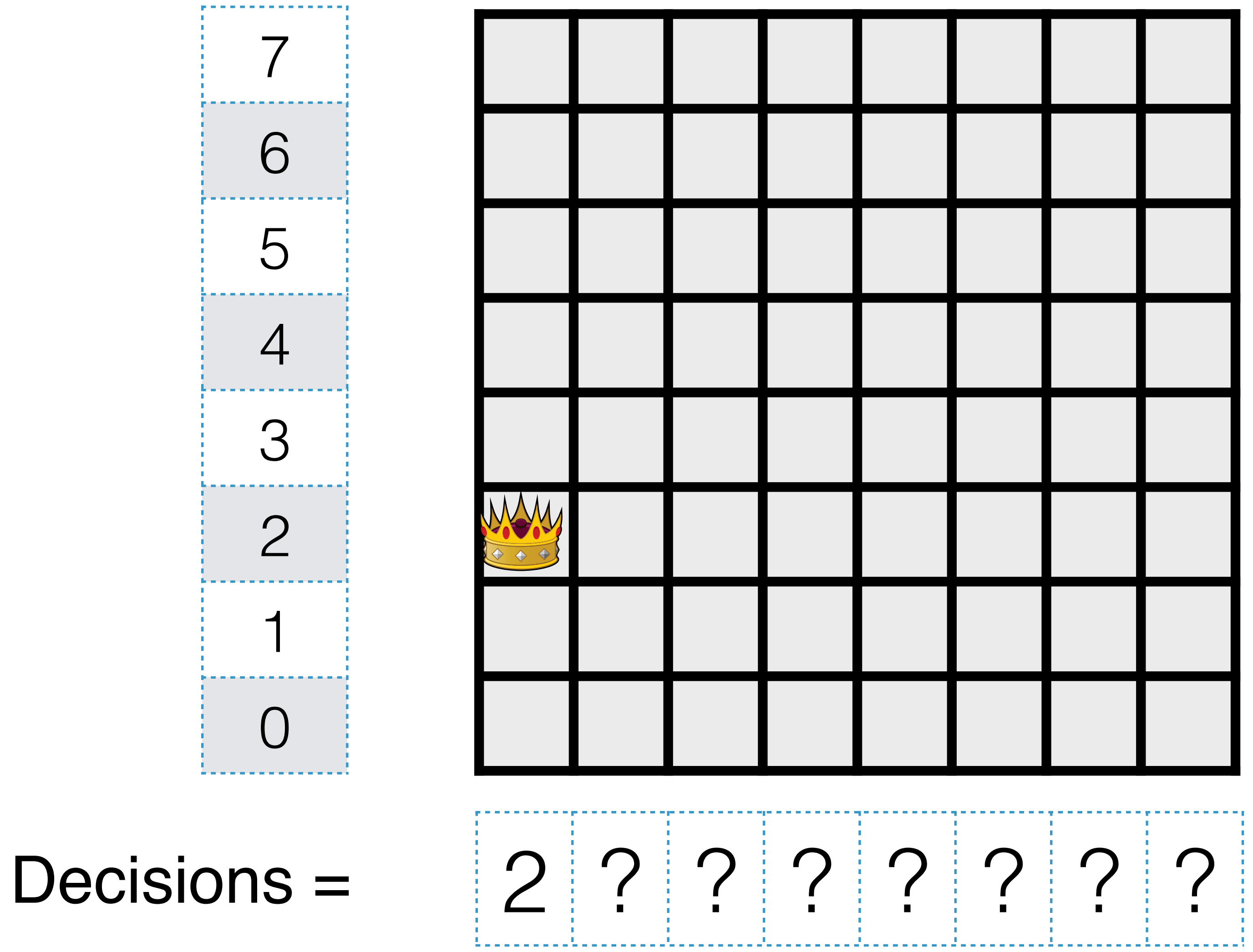
A boolean {True/False} for each cell telling whether or not a queen is present

Drawback: Require to test the three types of constraints: no two queens share the same row, column, or diagonal.

F	F	F	T	F	F	F	F
F	F	F	F	F	T	F	F
F	F	F	F	F	F	F	T
F	T	F	F	F	F	F	F
F	F	F	F	F	F	T	F
T	F	F	F	F	F	F	F
F	F	T	F	F	F	F	F
F	F	F	F	T	F	F	F

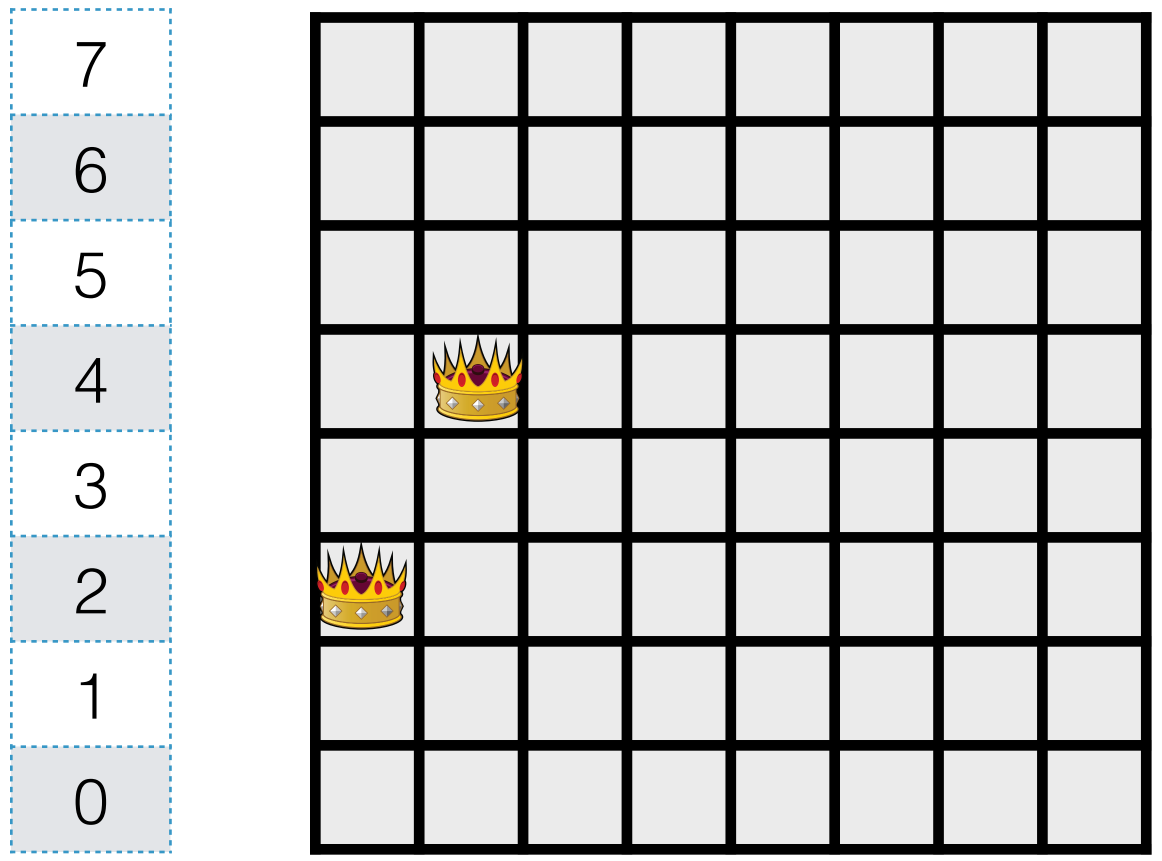
N-Queens: modeling considerations

An integer for each column {0,…,N-1} telling in which row to place the queen



N-Queens: modeling considerations

An integer for each column {0,…,N-1} telling in which row to place the queen

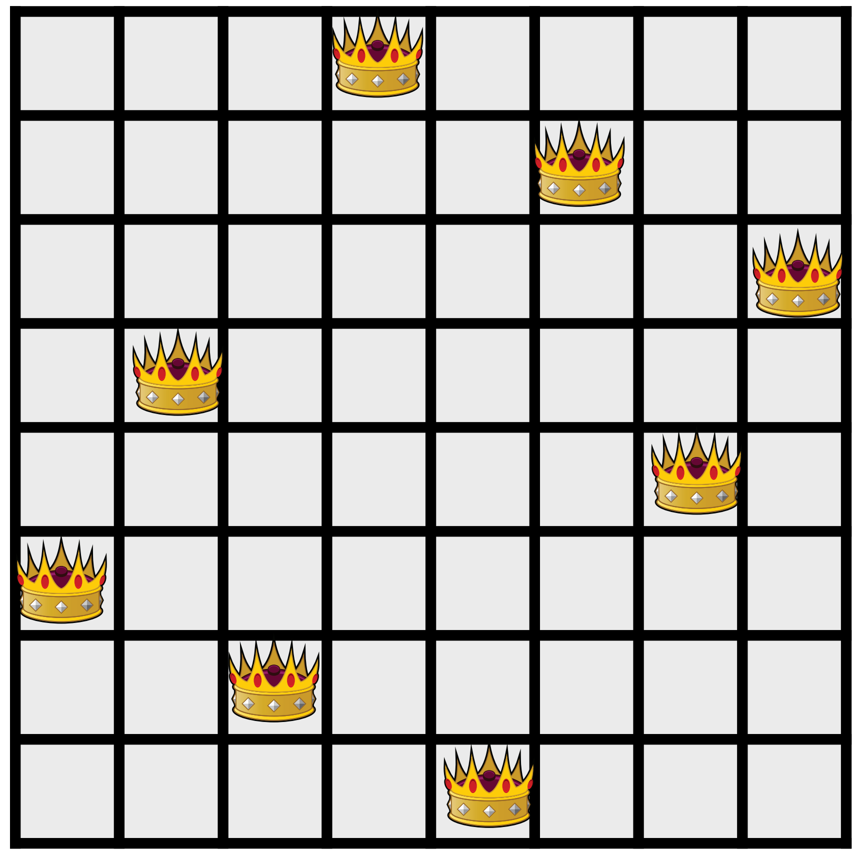


Decisions = | 2 | 4 | ? | ? | ? | ? | ? | ? |

N-Queens: modeling considerations

An integer for each column {0,…,N-1} telling in which row to place the queen

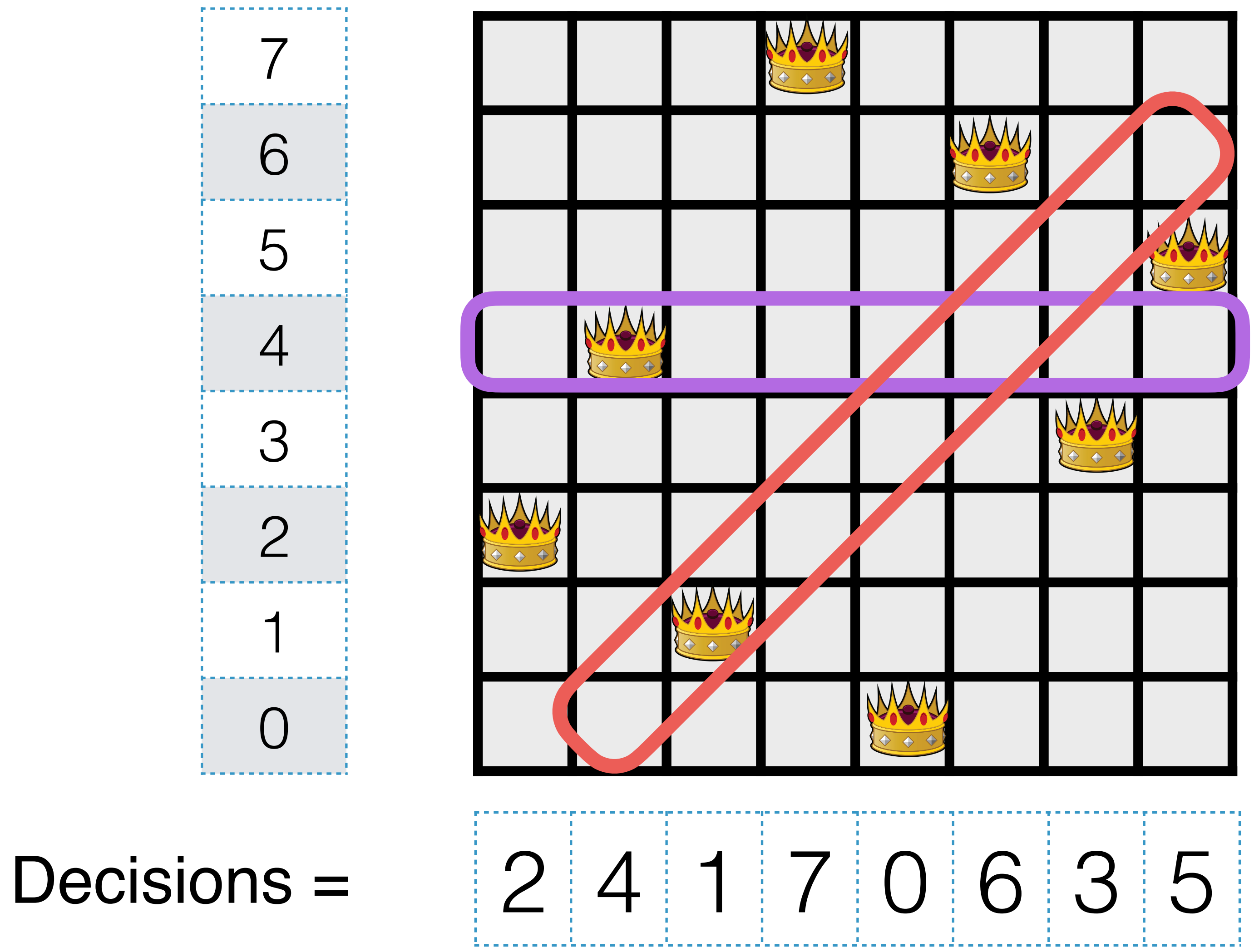
7
6
5
4
3
2
1
0



Decisions =	2	4	1	7	0	6	3	5

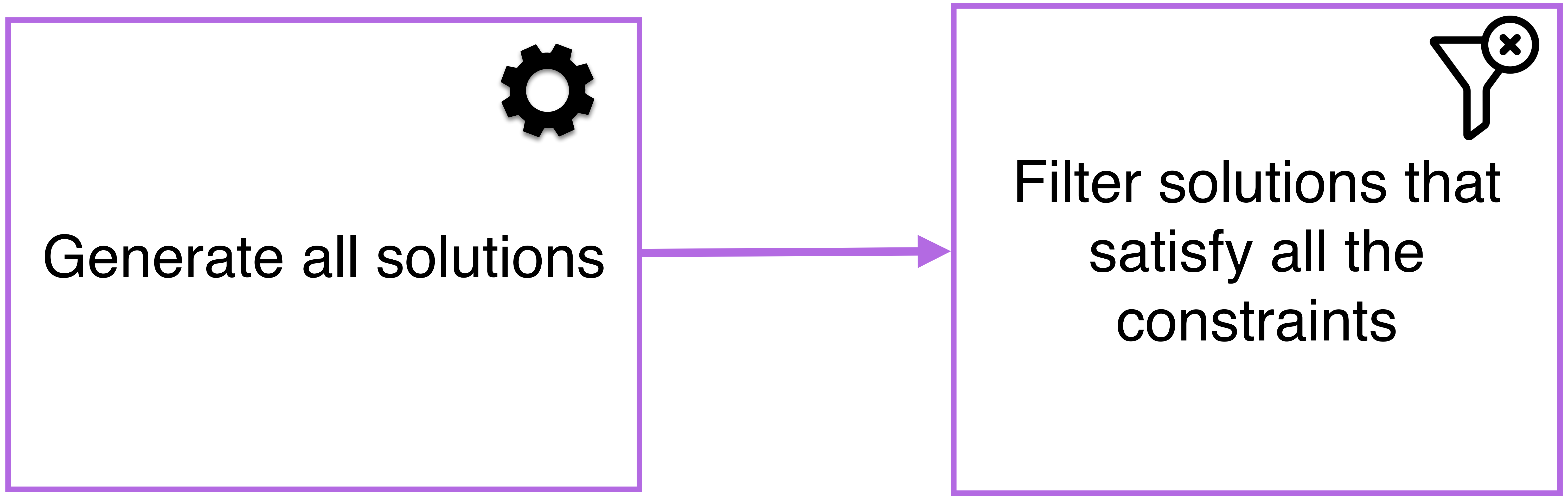
N-Queens: modeling considerations

Advantage: only two types of constraints: no two queens share the same row, ~~column~~, or diagonal.



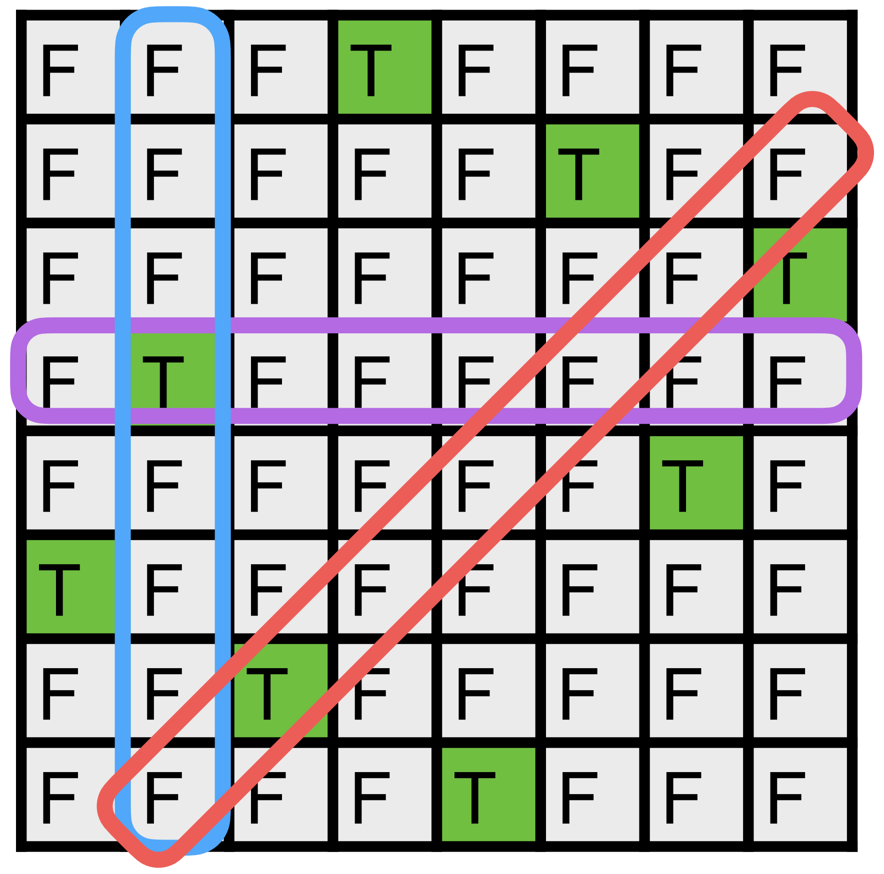
Discovering all the solutions to a CSP

- Let us make it generic

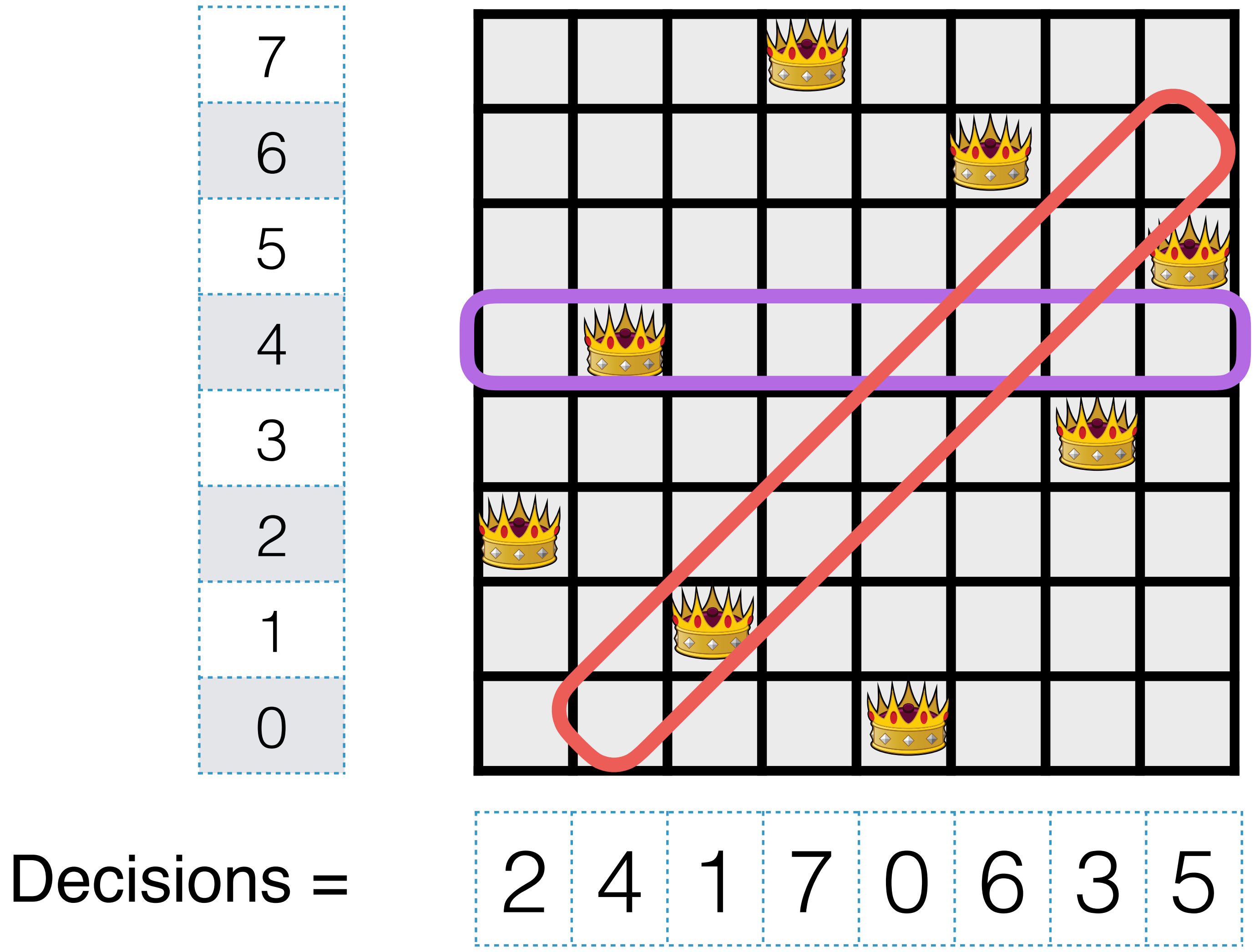


Number of solutions in our two models

2^{64}



$8^8 = 2^{24}$



Generate all the solutions …

- Backtracking Depth First Search

```java
public class NQueensChecker {

    int [] q;
    int n = 0;

    public NQueensChecker(int n) {
        this.n = n;
        q = new int[n];
    }

    public void dfs() {
        dfs(0);
    }

    private void dfs(int idx) {
       if (idx == n) {
            // candidate solution
        } else {
            for (int i = 0; i < n; i++) {
                q[idx] = i;
                dfs(idx+1, onSolution);
            }
        }
    }

}
```

… and filter them

- Backtracking Depth First Search + Filter

```java
public class NQueensChecker {

    int [] q;
    int n = 0;

    public NQueensChecker(int n) {
        this.n = n;
        q = new int[n];
    }

    public void dfs() {
        dfs(0);
    }

    private void dfs(int idx) {
       if (idx == n) {
           if (constraintsSatisfied()) {
               // output solution
           }
       } else {
           for (int i = 0; i < n; i++) {
               q[idx] = i;
               dfs(idx+1);
           }
       }
    }

}
```

```java
public boolean constraintsSatisfied() {
    for (int i = 0; i < n; i++) {
        for (int j = i+1; j < n; j++) {
            // no two queens on the same row
            if (q[i] == q[j]) return false;
            // no two queens on the diagonal
            if (Math.abs(q[j] - q[i]) == j-i) {
                return false;
            }
        }
    }
    return true;
}
```

Notice that this approach is quite generic. You just need a method (could be made abstract) to check the constraints ✅

"Hollywood Principle: Don't call us, we'll call you"

```java
public static void main(String[] args) {
    NQueensChecker q = new NQueensChecker(8);
    ArrayList<int []> solutions = new ArrayList<>();

    q.dfs(0, solution -> solutions.add(solution));
}
```

```java
import java.util.function.Consumer;

public class NQueensChecker {

    int [] q;
    int n = 0;

    public NQueensChecker(int n) {
        this.n = n;
        q = new int[n];
    }

    public void dfs(Consumer<int []> onSolution) {
        dfs(0,onSolution);
    }

    private void dfs(int idx, Consumer<int []> onSolution) {
       if (idx == n) {
            if (constraintsSatisfied()) {
               onSolution.accept(Arrays.copyOf(q,n));
            }
        } else {
            for (int i = 0; i < n; i++) {
                q[idx] = i;
                dfs(idx+1, onSolution);
            }
        }
    }
}
```

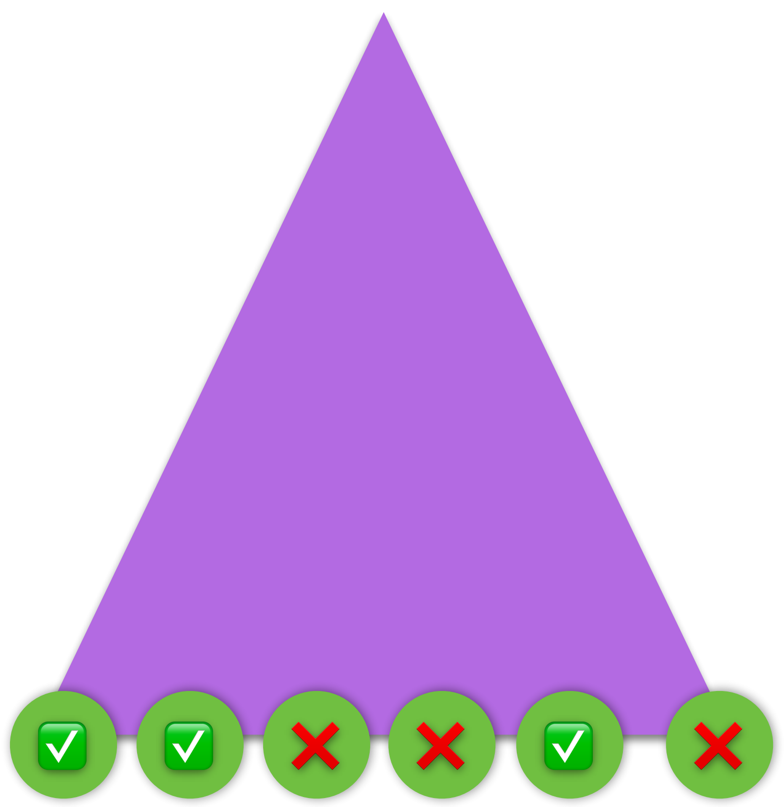
```java
@FunctionalInterface
public interface Consumer<T> {
    void accept(T t);
}
```

Demo

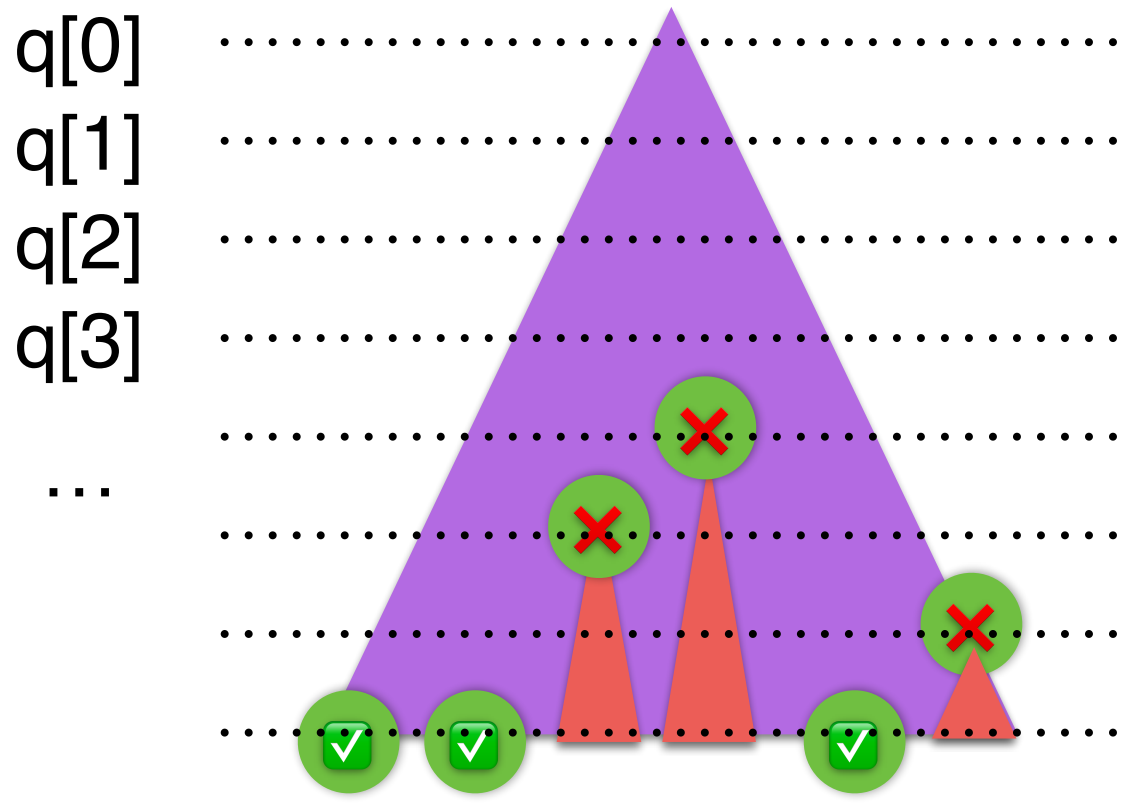
DFS + Prune

Principle

- DFS + filter: only verify constraints when all the decisions are finished



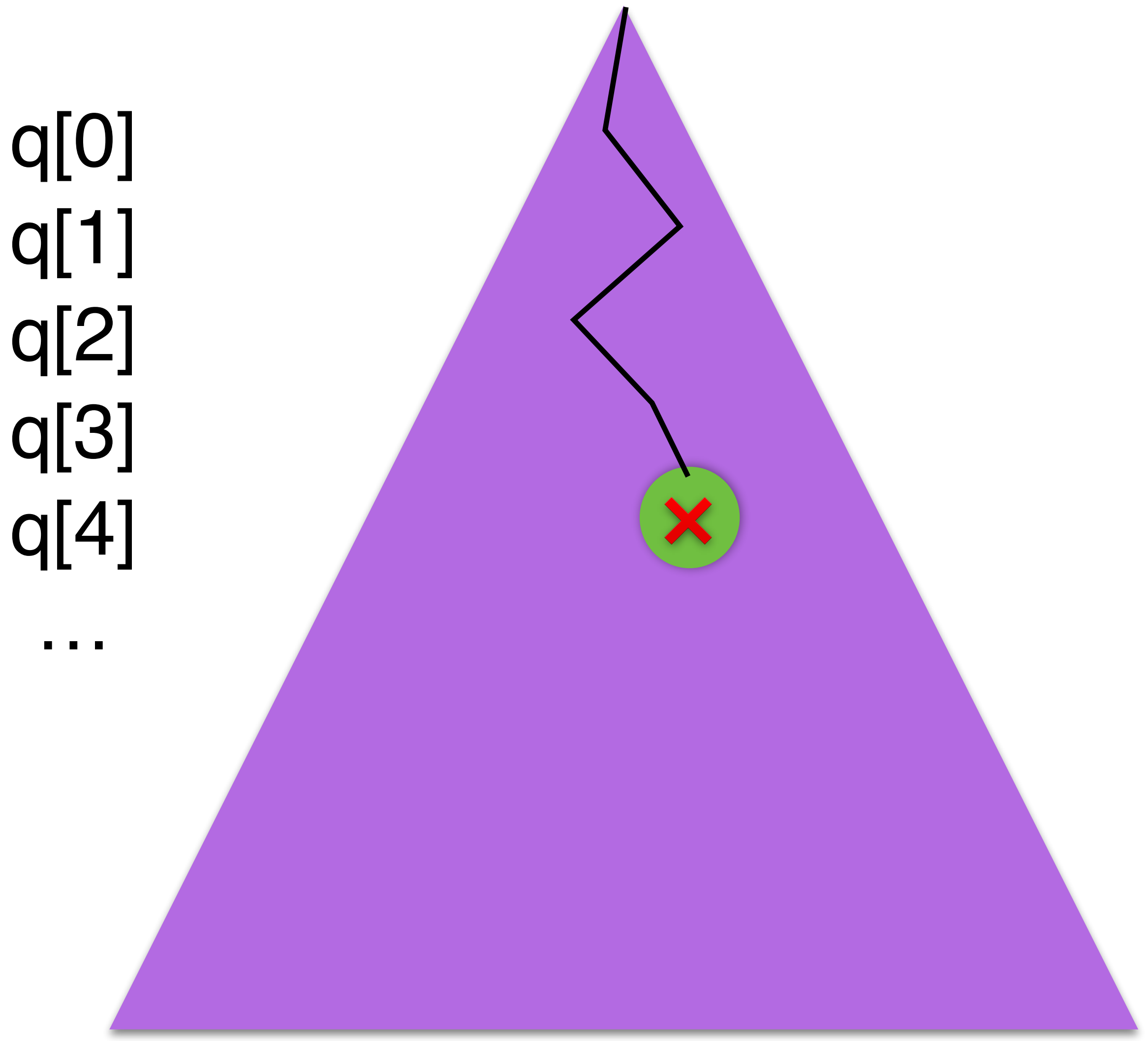
- DFS + Prune: verify constraints on a prefix of decisions (partial solution)

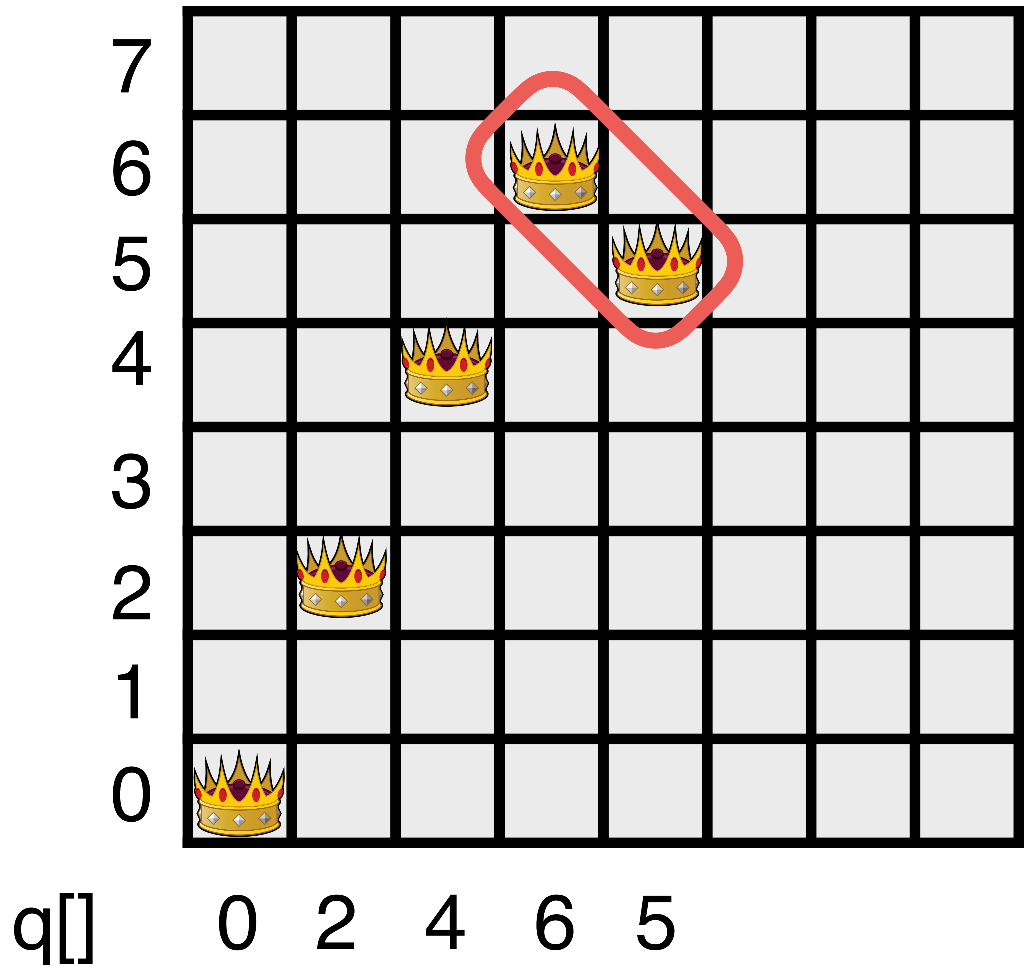


DFS+Prune

```java
public class NQueensPrune {

    int [] q;
    int n = 0;


    public NQueensPrune(int n) {
        this.n = n;
        q = new int[n];
    }

    public void dfs(Consumer<int []> onSolution) {
        dfs(0,onSolution);
    }

    private void dfs(int idx, Consumer<int []> onSolution) {
        if (idx == n) {
            onSolution.accept(Arrays.copyOf(q, n));
        } else {
            for (int i = 0; i < n; i++) {
                q[idx] = i;
                if (constraintsSatisfied(idx))
                    dfs(idx + 1, onSolution);
            }
        }
    }

}
```

```java
    public boolean constraintsSatisfied(int j) {
        for (int i = 0; i < j; i++) {
            // no two queens on the same row
            if (q[i] == q[j]) return false;
            // no two queens on the diagonal
            if (Math.abs(q[j] - q[i]) == j - i) {
                return false;
            }
        }
        return true;
    }
```





Drawback of DFS+Prune

- Search per level
  - The backtracking works with only one index “i” because you overwrite previous decisions
- Only one set of decision variables
- Only one inference hardcoded and problem specific, none of the code is reusable for solving another problem, even quite similar (let’s say SUDOKU)
- Our next version will target genericity and reusability of ingredients

Tiny-CSP Model

N-Queens Model with Tiny-CSP

```java
int n = 10;
TinyCSP csp = new TinyCSP();
Variable[] q = new Variable[n];

for (int i = 0; i < n; i++) {
    q[i] = csp.makeVariable(n);
}

for (int i = 0; i < n; i++) {
    for (int j = i+1; j < n; j++) {
        // queens q[i] and q[i] not on ...
        csp.notEqual(q[i],q[j],0); // ... the same line
        csp.notEqual(q[i],q[j],i-j); // ... the same left diagonal
        csp.notEqual(q[i],q[j],j-i); // ... the same right diagonal
    }
}

ArrayList<int []> solutions = new ArrayList<>();
// collect all the solutions
csp.dfs(solution -> {
    solutions.add(solution);
});
```

Variables

Constraints

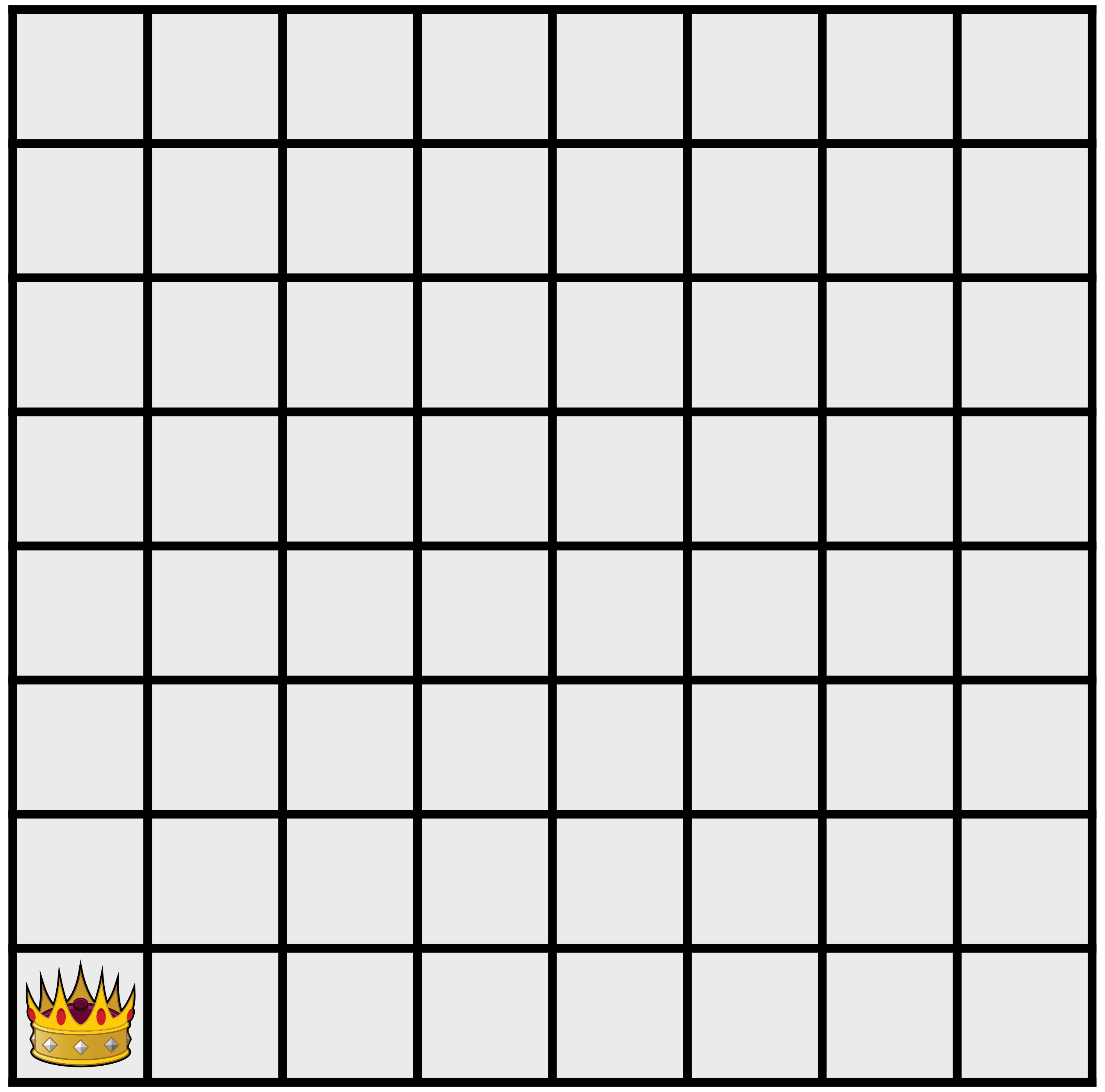
Search

Let’s make this work …

N-Queens: Model in MiniCP

- Representation = a model:
  - Holds an array of integer variables with one variable per column.

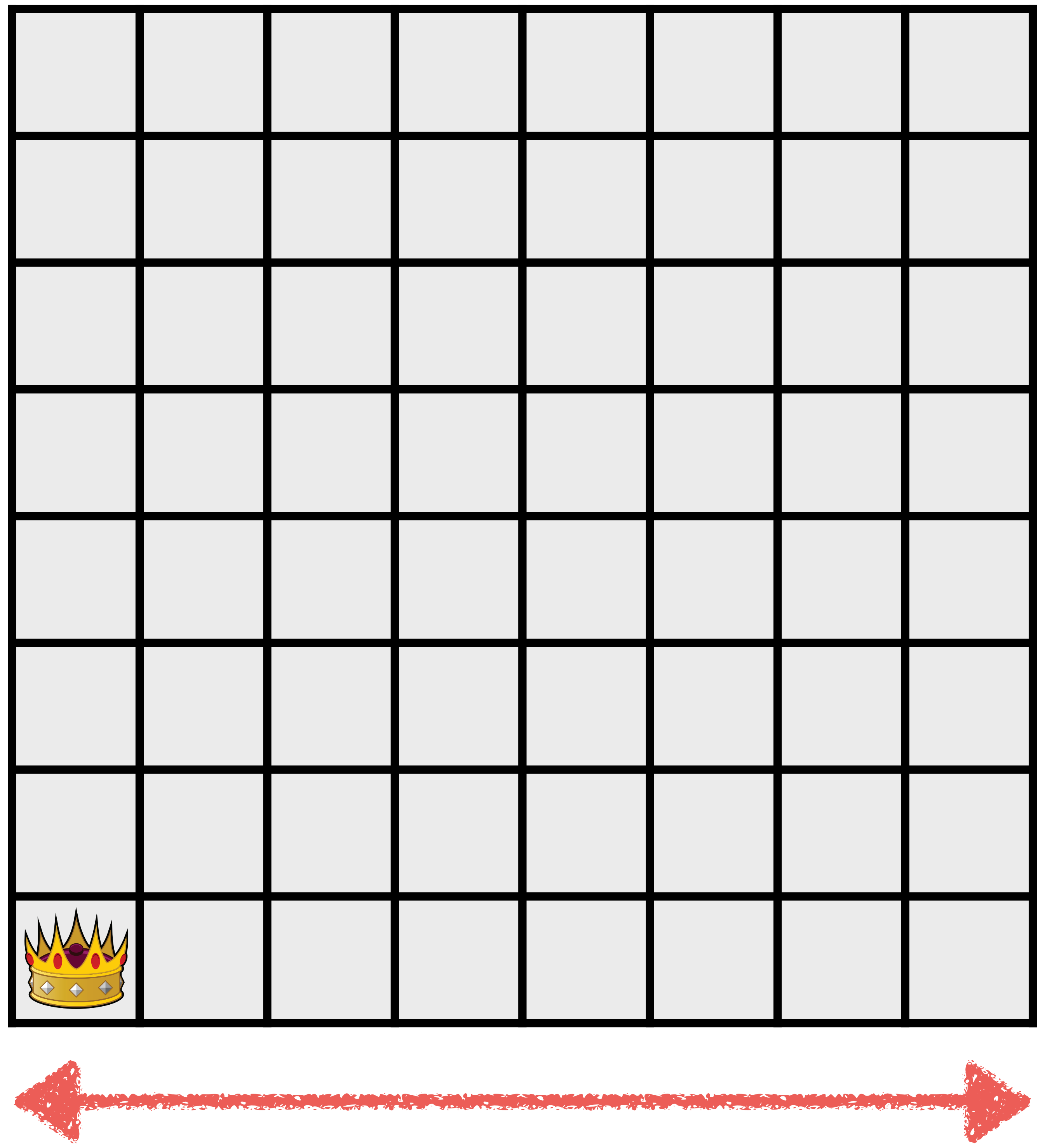
```java
int n = 8;
TinyCSP csp = new TinyCSP();
Variable[] q = new Variable[n];

for (int i = 0; i < n; i++) {
    q[i] = csp.makeVariable(n);
}
```



8-Queens: Model in MiniCP

- Representation = a model:
  - Holds an array of integer variables with one variable per column.

```
int n = 8;
TinyCSP csp = new TinyCSP();
Variable[] q = new Variable[n];

for (int i = 0; i < n; i++) {
    q[i] = csp.makeVariable(n);
}
```

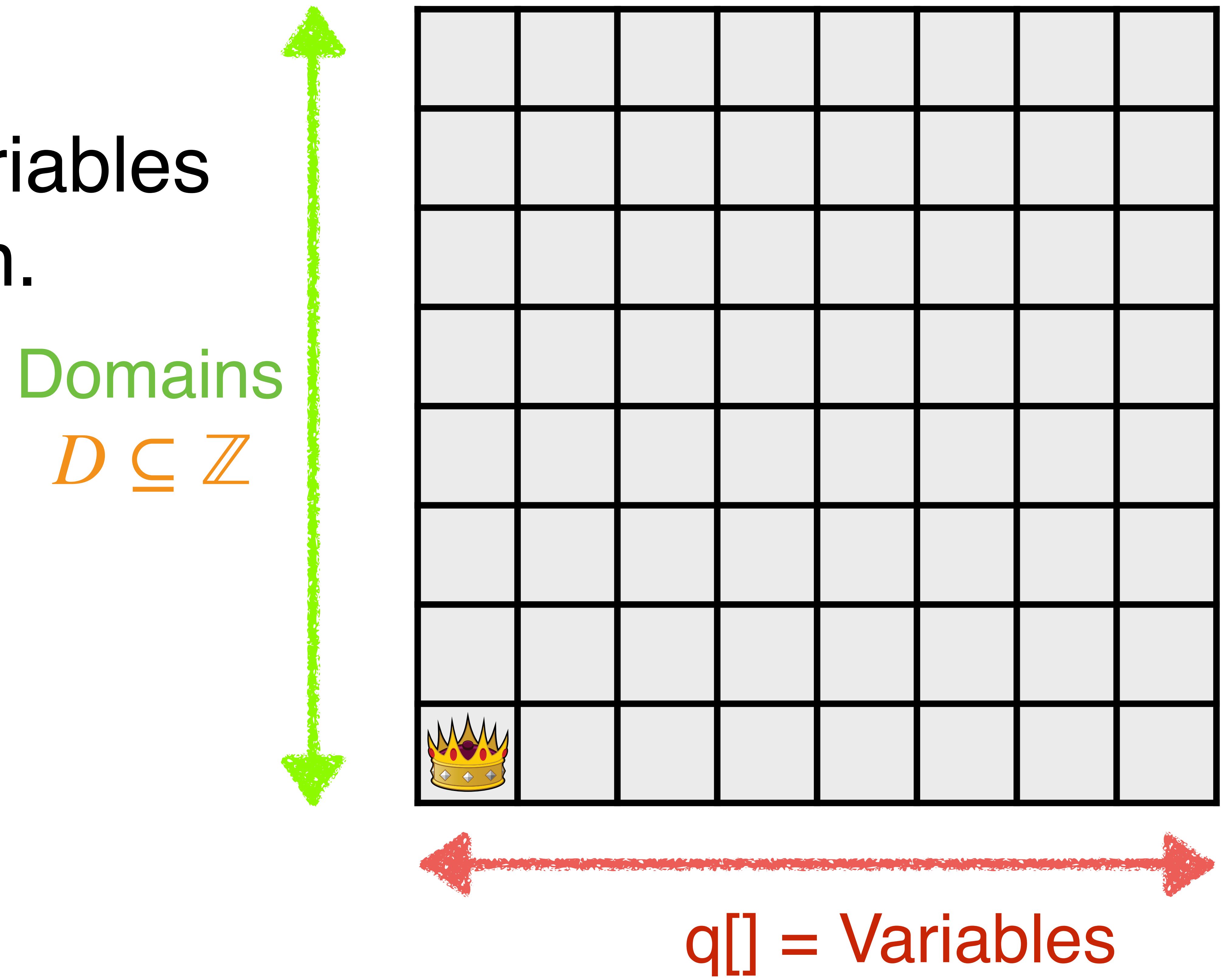


q[] = Variables

8-Queens: Model in MiniCP

- Representation = a model:
  - Holds an array of integer variables with one variable per column.

```java
int n = 8;
TinyCSP csp = new TinyCSP();
Variable[] q = new Variable[n];

for (int i = 0; i < n; i++) {
    q[i] = csp.makeVariable(n);
}
```



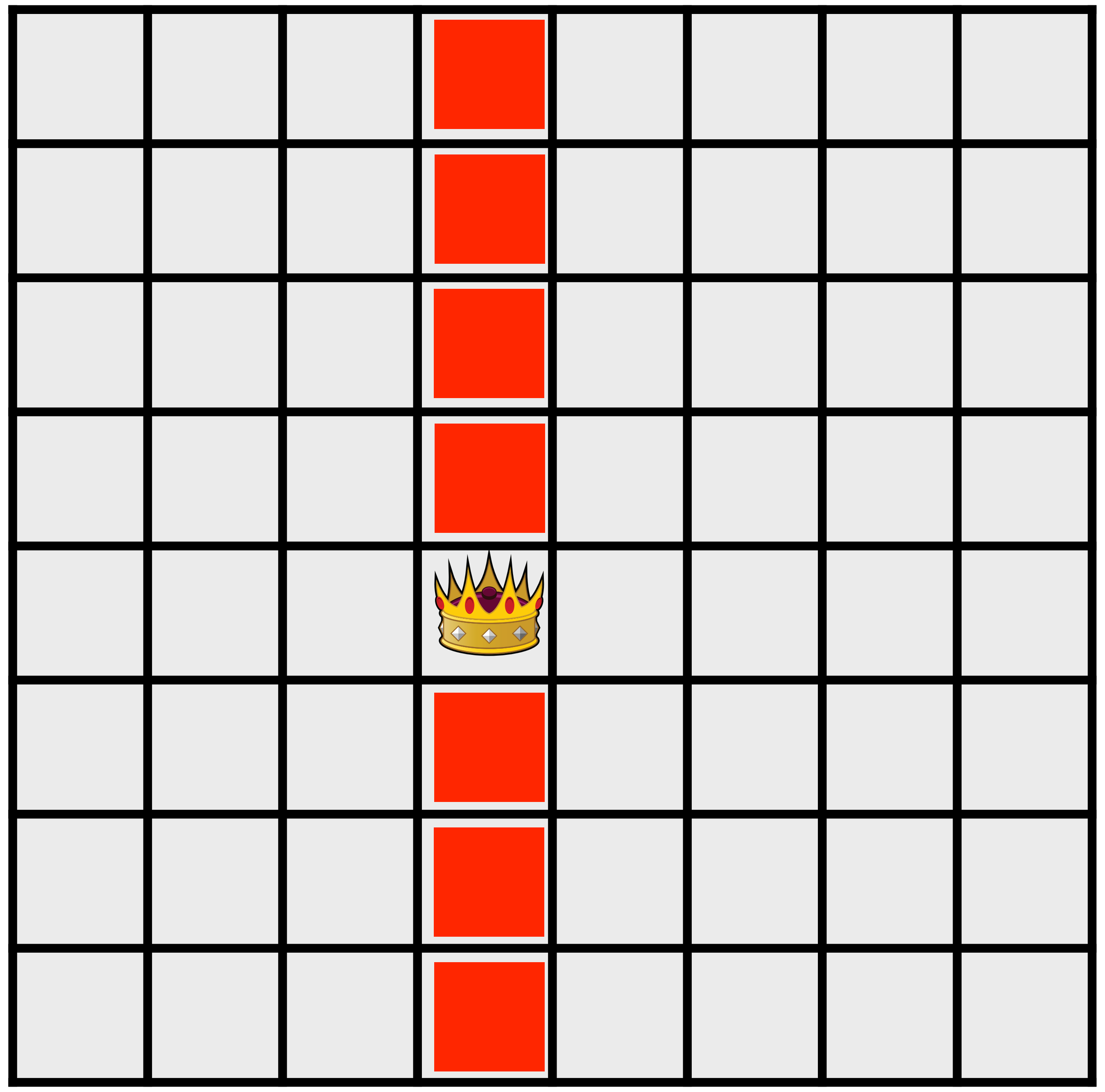
8-Queens: Model in MiniCP

- Representation = a model:
  - Holds an array of integer variables with one variable per column.

```
int n = 10;
TinyCSP csp = new TinyCSP();
Variable[] q = new Variable[n];

for (int i = 0; i < n; i++) {
    q[i] = csp.makeVariable(n);
}
```

  - Cannot be in the same column…

```
for (int i = 0; i < n; i++) {
    for (int j = i+1; j < n; j++) {




    }
}
```



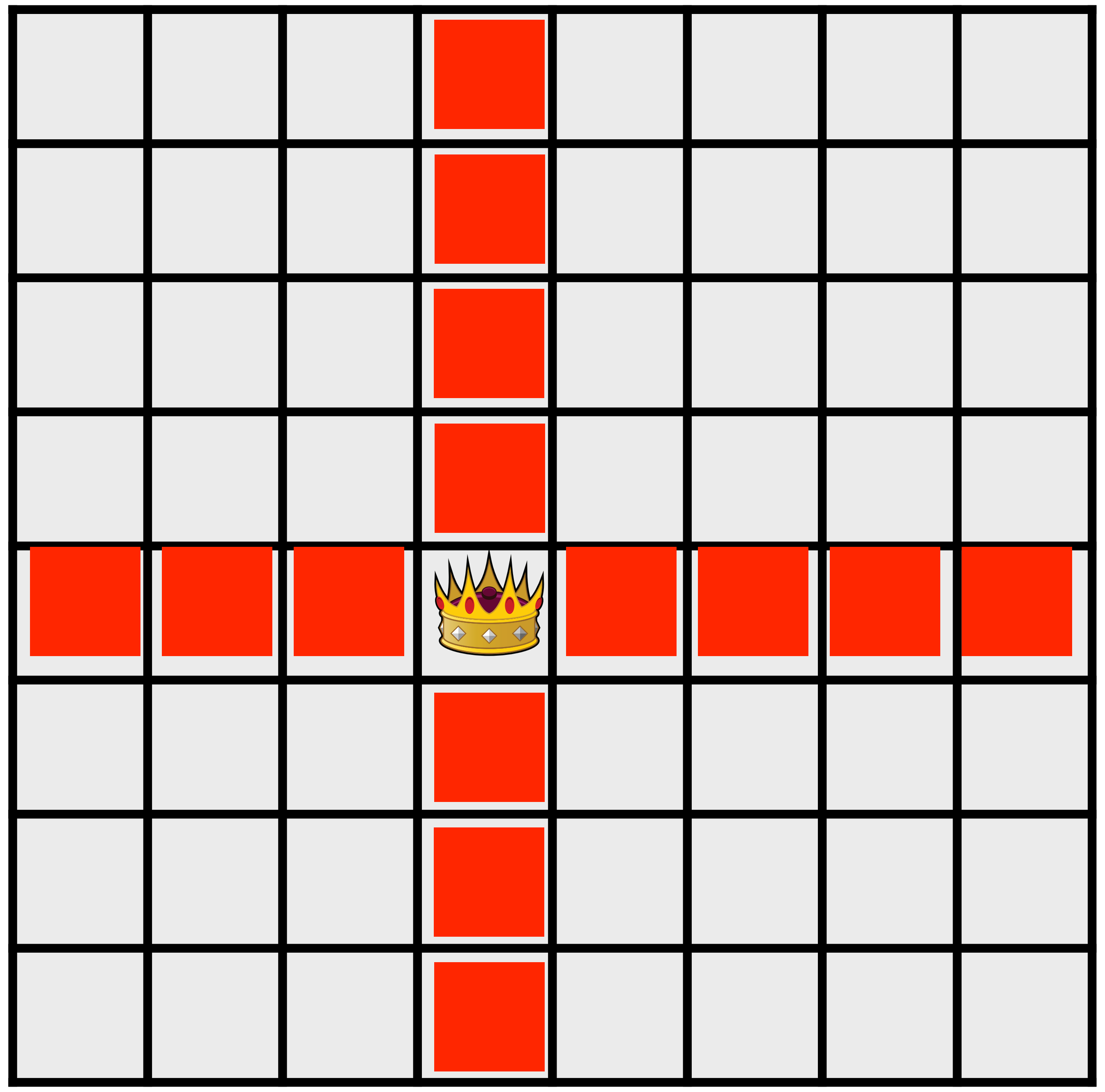
8-Queens: Model in MiniCP

- Representation = a model:
  - Holds an array of integer variables with one variable per column

```
int n = 10;
TinyCSP csp = new TinyCSP();
Variable[] q = new Variable[n];

for (int i = 0; i < n; i++) {
    q[i] = csp.makeVariable(n);
}
```

  - Cannot be on the same row…

```
for (int i = 0; i < n; i++) {
    for (int j = i+1; j < n; j++) {
      // queens q[i] and q[i] not on ...
      csp.notEqual(q[i],q[j],0); // line


    }
}
```



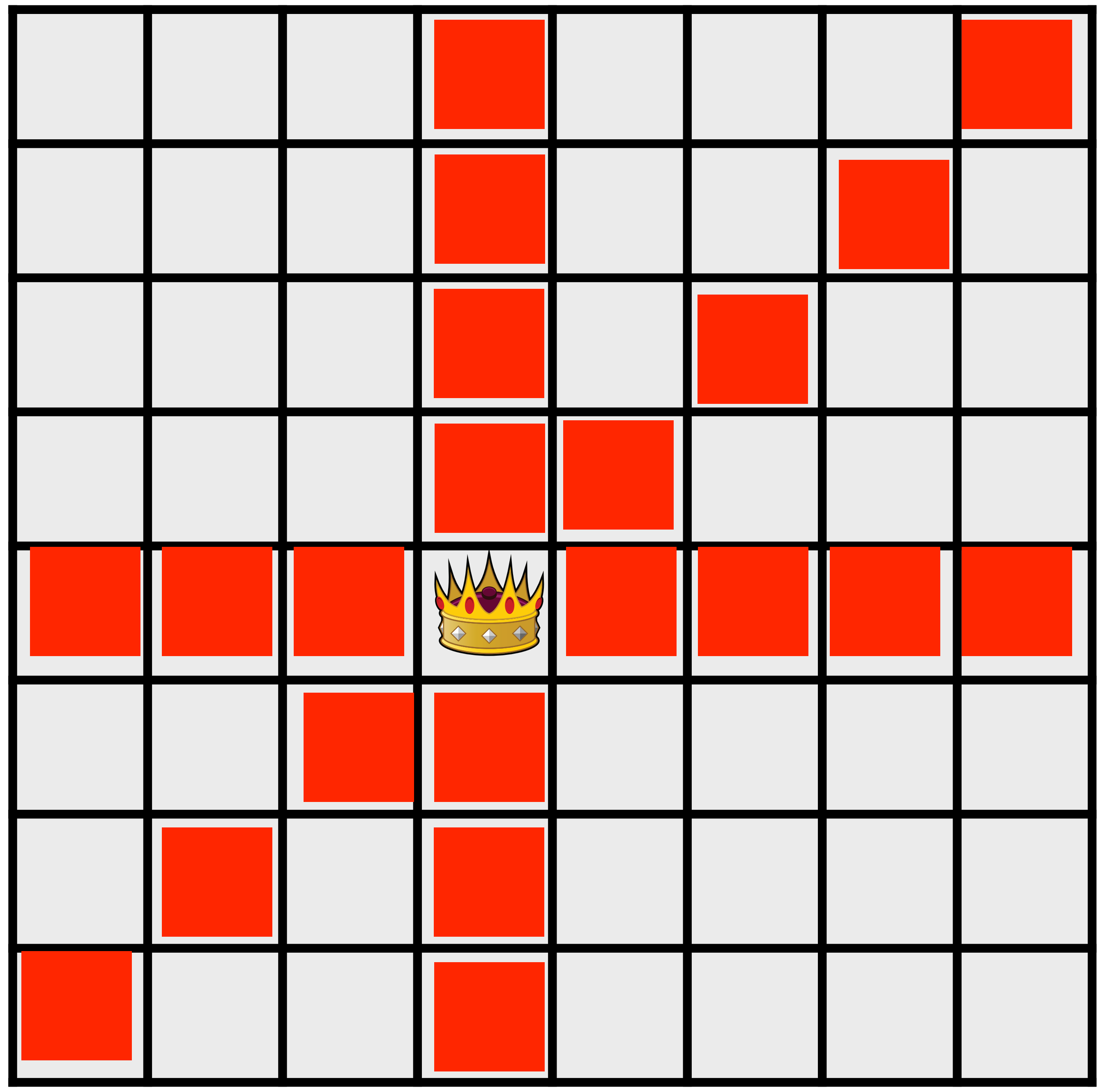
8-Queens: Model in MiniCP

- Representation = a model:
  - Holds an array of integer variables with one variable per column.

```
int n = 10;
TinyCSP csp = new TinyCSP();
Variable[] q = new Variable[n];

for (int i = 0; i < n; i++) {
    q[i] = csp.makeVariable(n);
}
```

  - Cannot be on the same diagonal…

```
for (int i = 0; i < n; i++) {
    for (int j = i+1; j < n; j++) {
      // queens q[i] and q[i] not on ...
      csp.notEqual(q[i],q[j],0); // line
      csp.notEqual(q[i],q[j],i-j); // left diagonal

   }
}
```



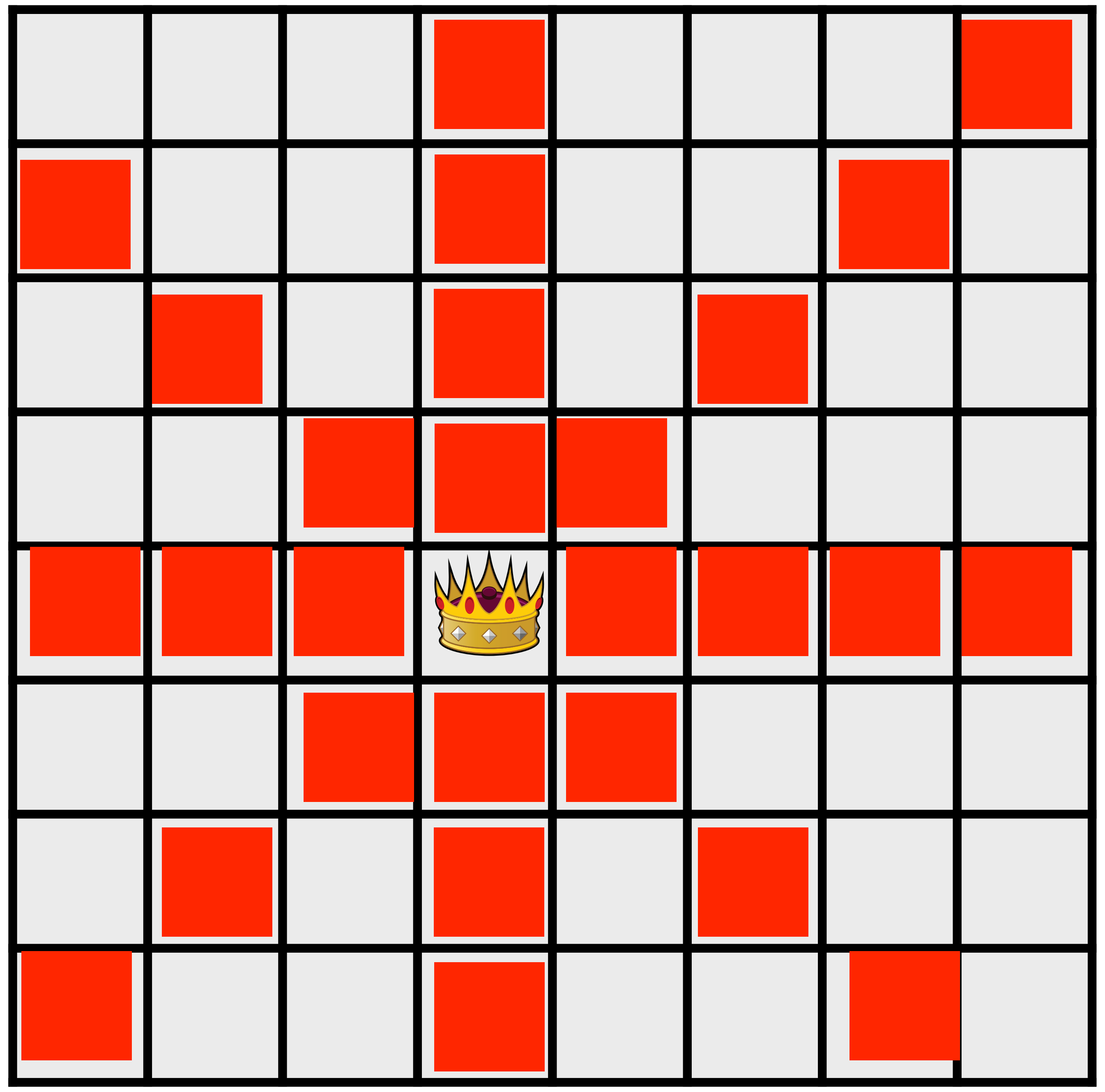
8-Queens: Model in MiniCP

- Representation = a model:
  - Holds an array of integer variables with one variable per column.

```
int n = 10;
TinyCSP csp = new TinyCSP();
Variable[] q = new Variable[n];

for (int i = 0; i < n; i++) {
    q[i] = csp.makeVariable(n);
}
```

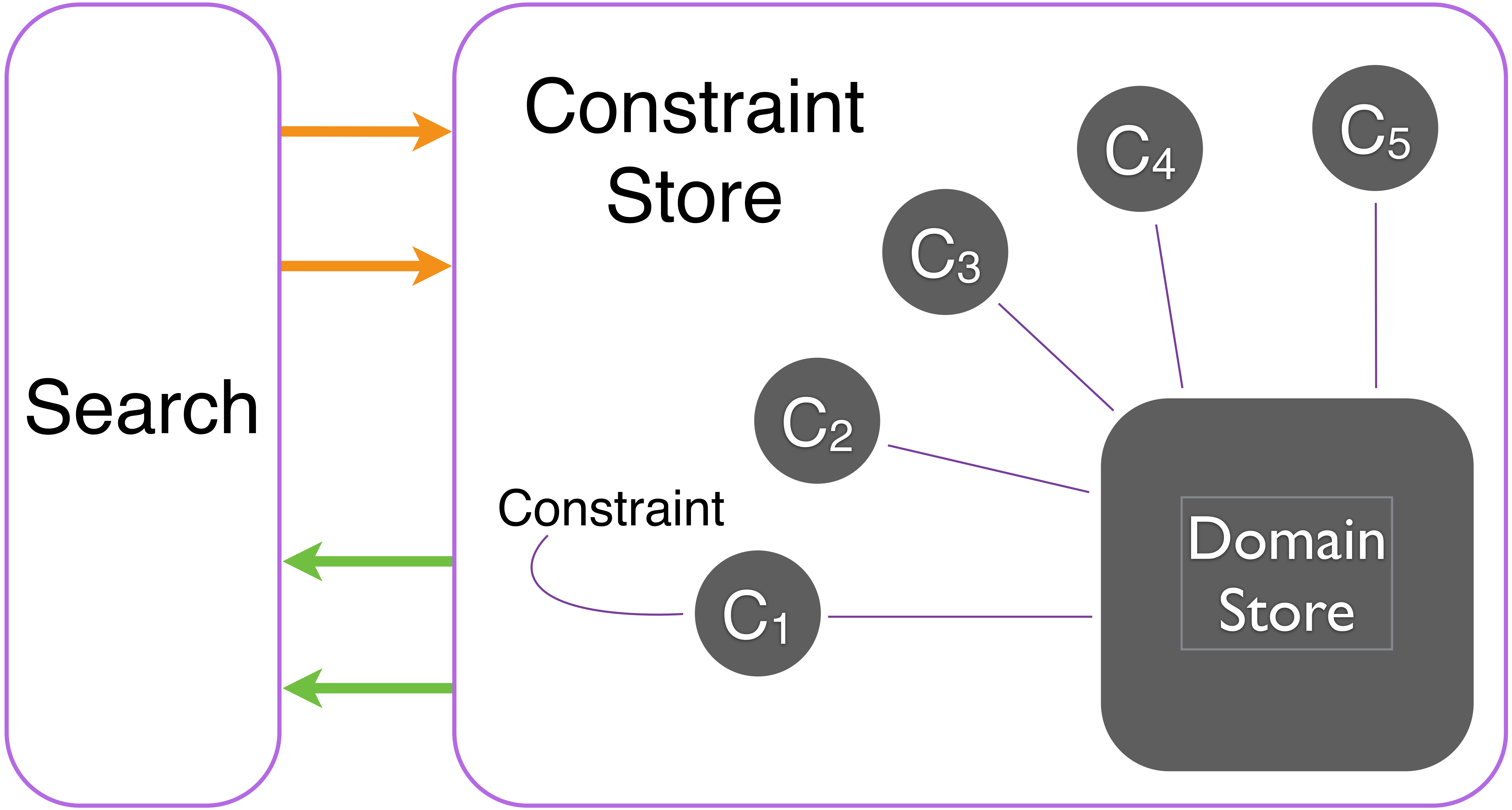
  - Cannot be on the same diagonals…

```
for (int i = 0; i < n; i++) {
    for (int j = i+1; j < n; j++) {
      // queens q[i] and q[i] not on ...
      csp.notEqual(q[i],q[j],0); // line
      csp.notEqual(q[i],q[j],i-j); // left diagonal
      csp.notEqual(q[i],q[j],j-i); // right diagonal
    }
}
```



Tiny-CSP Computation

Computational Paradigm



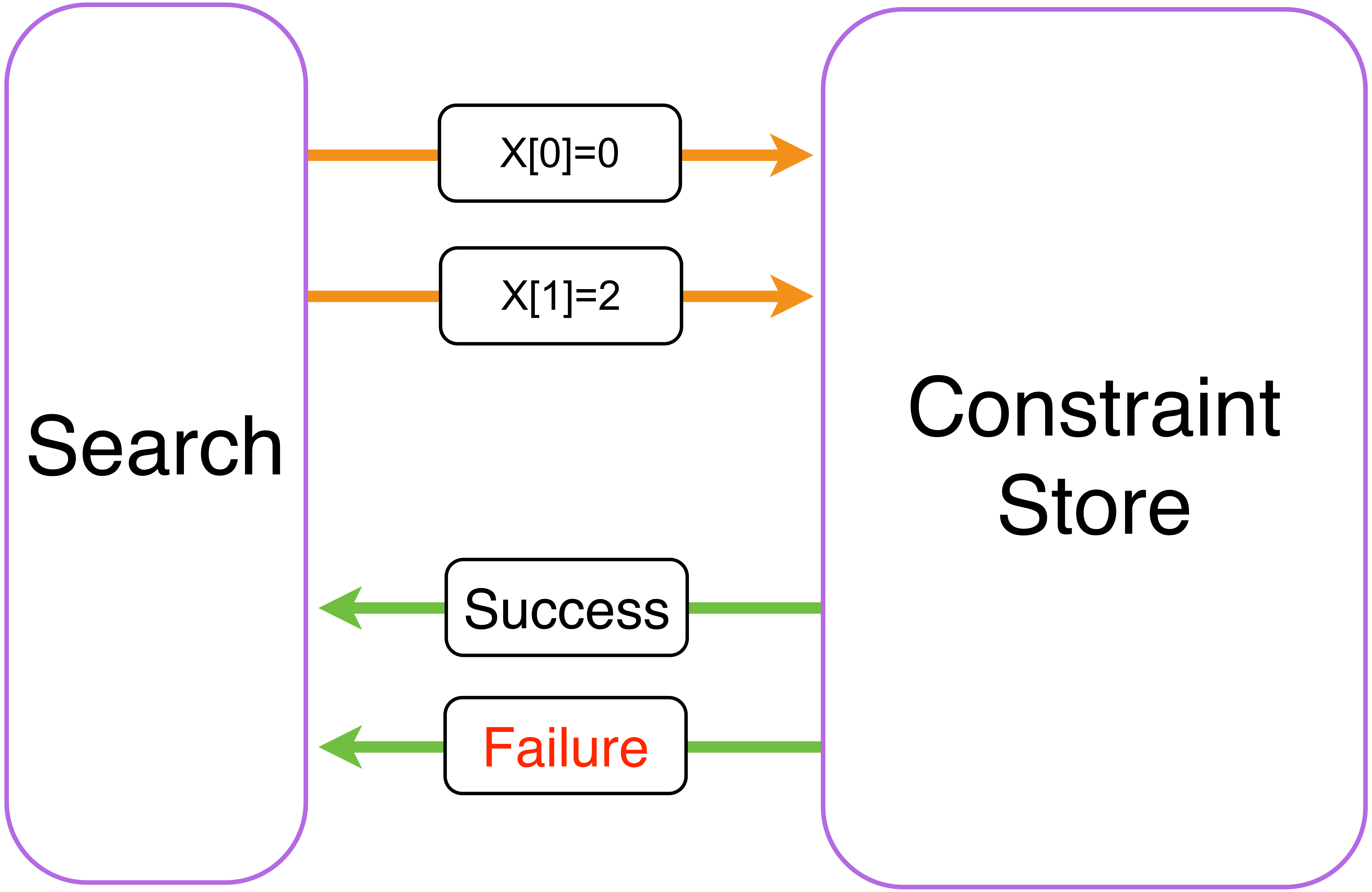
Computational Paradigm

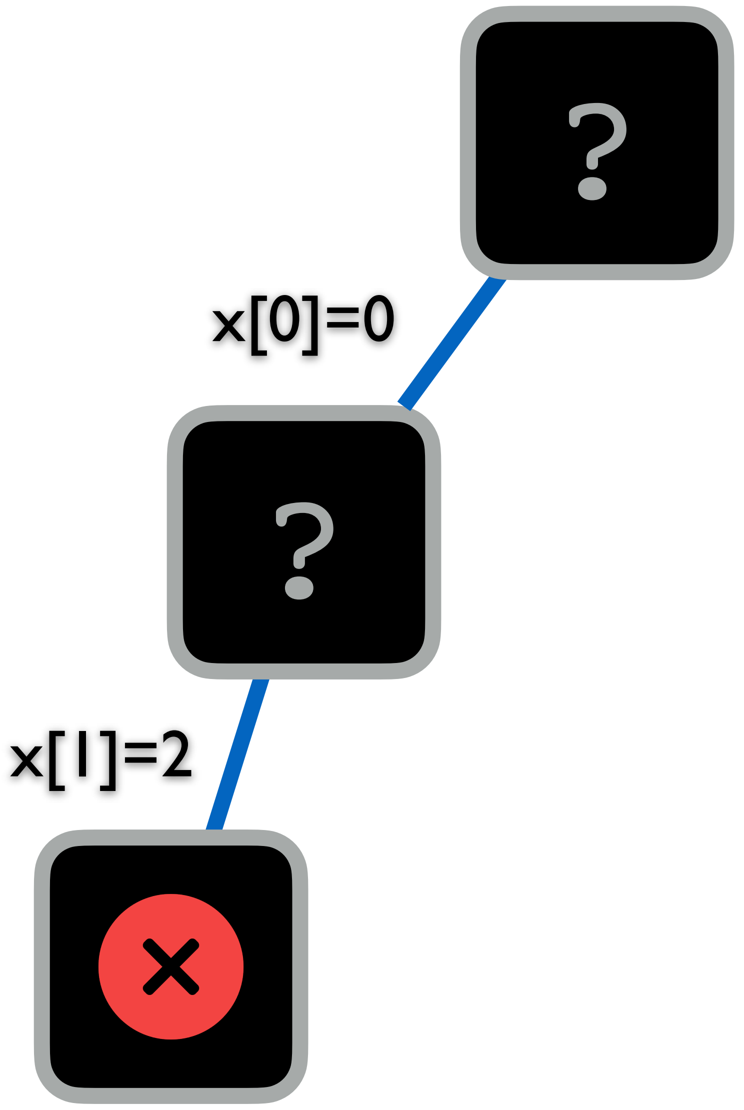
The propagation engine:

- This is the core of any constraint-programming solver.
- It is a simple fixpoint algorithm:

```
fixPoint()
{
  repeat
    select a constraint c;
    if c is infeasible given the domain store then
      return failure;
    else
      apply the pruning algorithm associated with c;
  until (no constraint can remove any value from the domain of its variables);
  return success;
}
```

Computational Paradigm





TinyCSP class

Constraint Store

```java
public class TinyCSP {

    List<Constraint> constraints = new LinkedList<>();
    List<Variable> variables = new LinkedList<>();

    public Variable makeVariable(int domSize) {
        Variable x = new Variable(domSize);
        variables.add(x);
        return x;
    }

    public void notEqual(Variable x, Variable y, int offset) {
        constraints.add(new NotEqual(x, y, offset));
        fixPoint();
    }

    public void fixPoint() {
        boolean fix = false;
        while (!fix) {
            fix = true;
            for (Constraint c : constraints) {
                fix &= !c.propagate();
            }
        }
    }
}
```

```java
abstract class Constraint {
    /**
     * Propagate the constraint and return
     * true if any value could be removed
     * @return true if at least one value of one
     *         variable could be removed
     */
    abstract boolean propagate();
}
```

```java
public class Variable {

    Domain dom;

    /**
     * Creates a variable with domain {0..n-1}     */
    public Variable(int n) {
        dom = new Domain(n);
    }
}
```

What does a constraint do ?

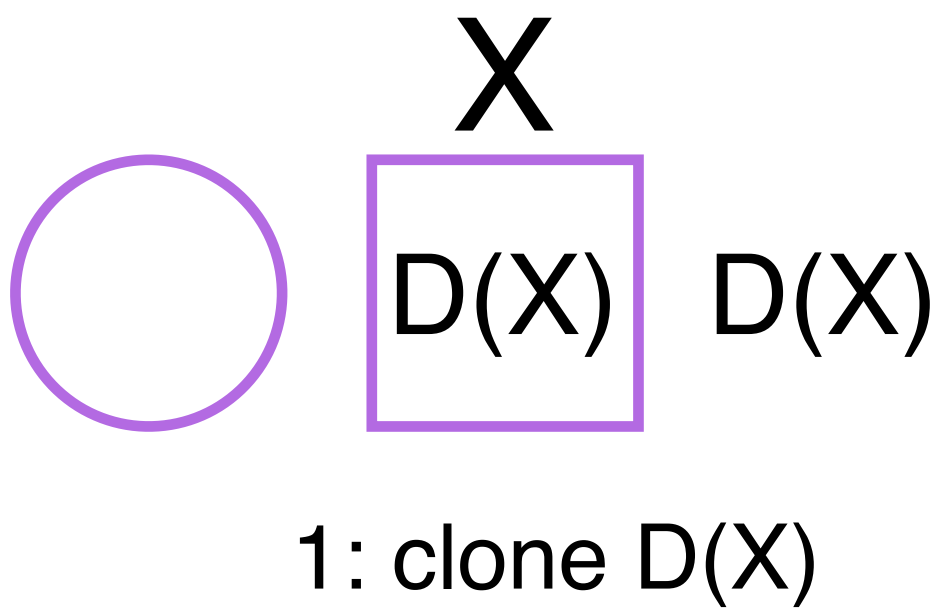
- Feasibility checking:
  - Can the constraint be satisfied given the values in the domains of its variables?
- Pruning:
  - If satisfiable = feasible, then a constraint removes values in the domains that cannot be part of any solution.

The Not Equal Constraint x != y + offset

```
class NotEqual extends Constraint {

    Variable x, y;
    int offset;

    public NotEqual(Variable x, Variable y, int offset) {
        this.x = x;
        this.y = y;
        this.offset = offset;
    }

    public NotEqual(Variable x, Variable y) {
        this(x, y, 0);
    }

    @Override
    boolean propagate() {
        if (x.dom.isFixed()) {
            return y.dom.remove(x.dom.min() - offset);
        }
        if (y.dom.isFixed()) {
            return x.dom.remove(y.dom.min() + offset);
        }
        return false;
    }
}
```

State Management

- When a value is removed it needs to be restored on backtrack
- TinyCSP will use a “backup” mechanism of the domains



State Management

- When a value is removed it needs to be restored on backtrack
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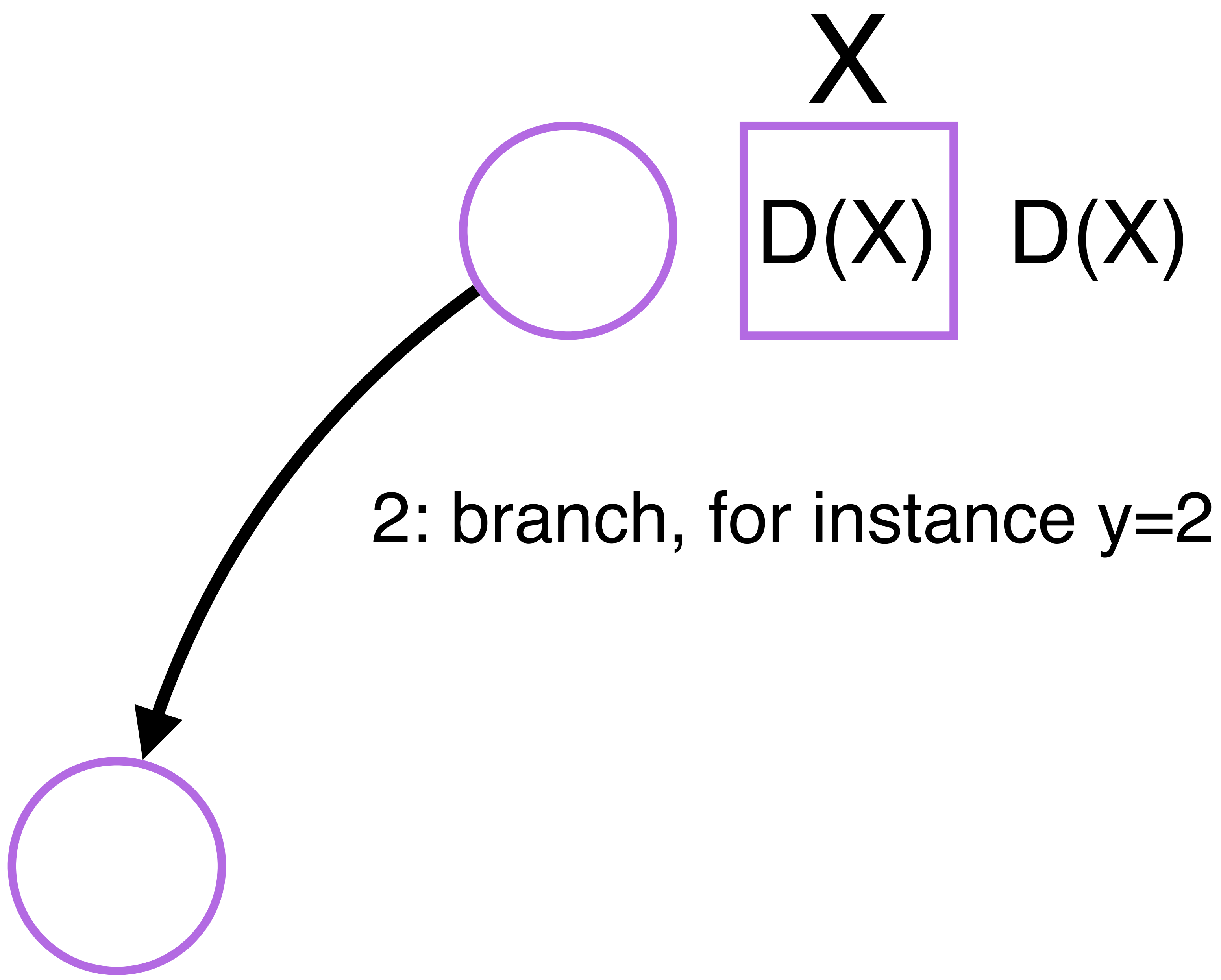

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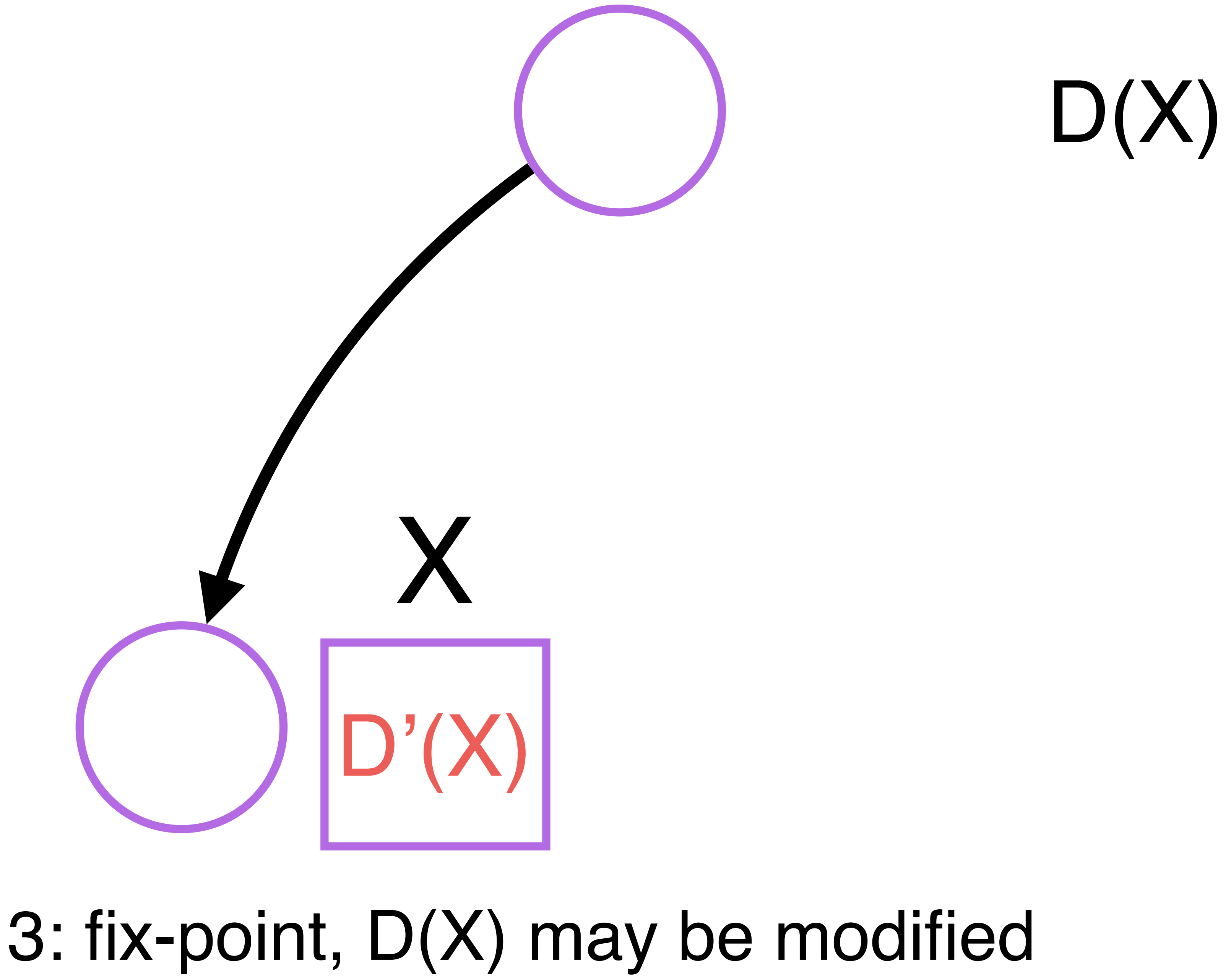

State Management

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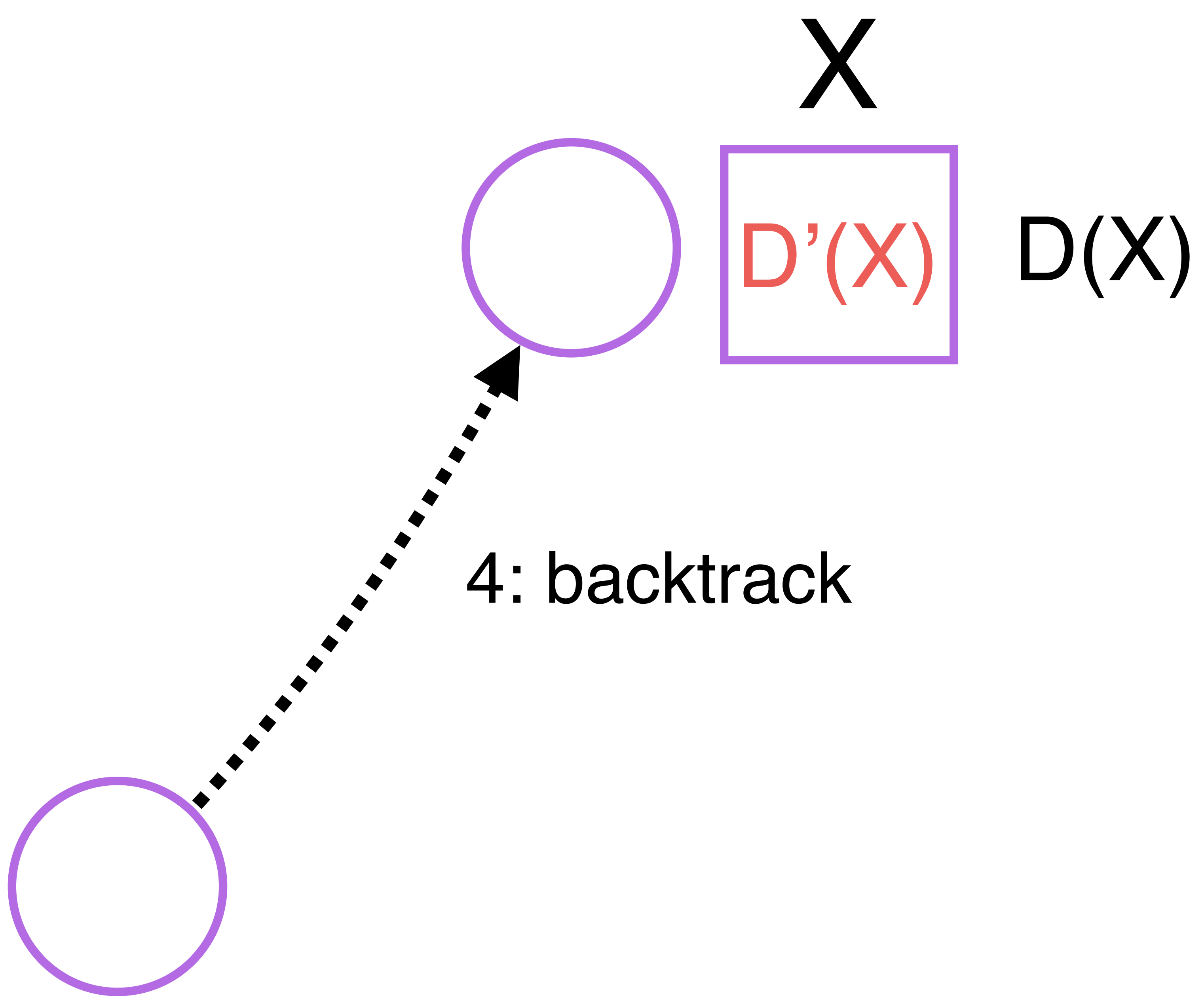


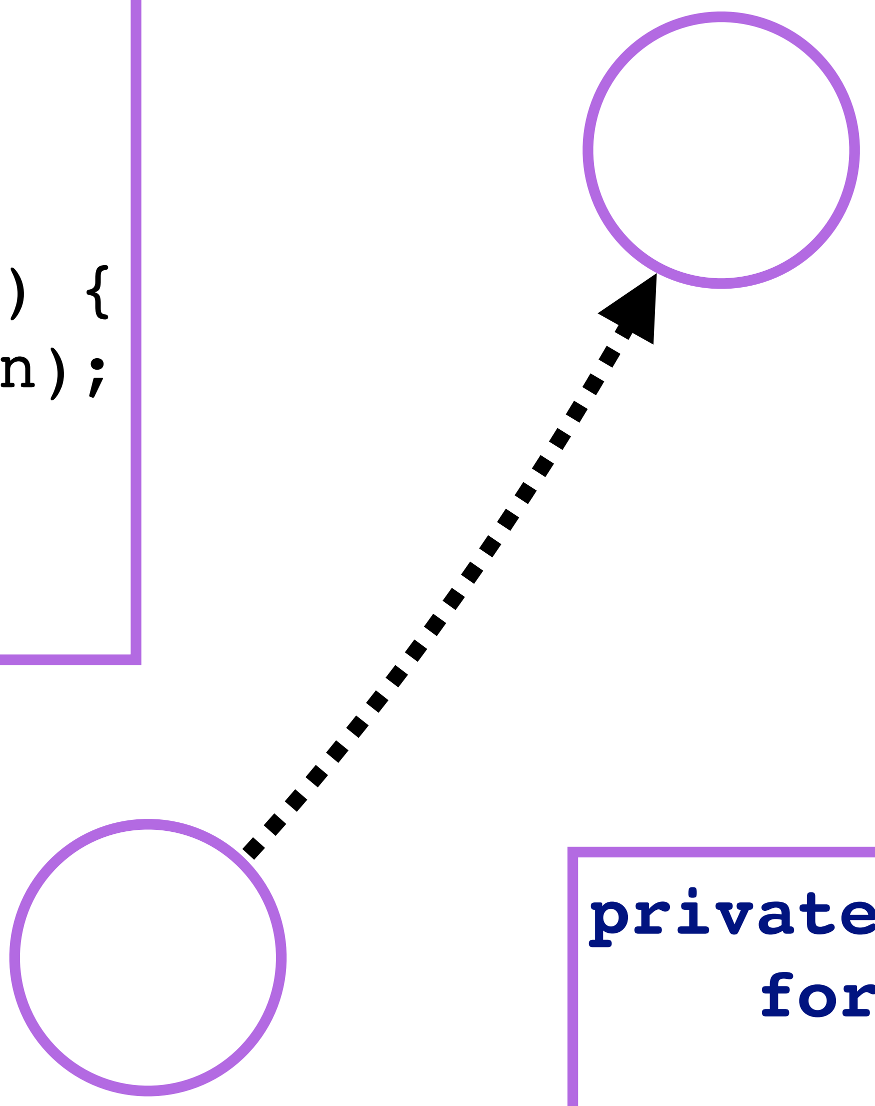
State Management

- When a value is removed it needs to be restored on backtrack
- TinyCSP will use a "backup" mechanism of the domains

```java
public class Variable {

    Domain dom;

    public Variable(int n) {
        dom = new Domain(n);
    }
}
```



X

D(X)

5: restore the cloned domain

```java
private void restoreDomains(ArrayList<Domain> backup) {
    for (int i = 0; i < variables.size(); i++) {
        variables.get(i).dom = backup.get(i);
    }
}
```

DFS

```java
public void dfs(Consumer<int[]> onSolution) {
    // pickup a variable that is not yet fixed if any
    Optional<Variable> notFixed = firstNotFixed();
    if (!notFixed.isPresent()) { // all variables fixed, a solution is found
        int[] solution = variables.stream().mapToInt(x -> x.dom.min()).toArray();
        onSolution.accept(solution);
    } else {
        Variable y = notFixed.get(); // take the unfixed variable
        int v = y.dom.min();
        ArrayList<Domain> backup = backupDomains();
        // left branch x = v
        try {
            y.dom.fix(v);
            fixPoint();
            dfs(onSolution);
        } catch (Inconsistency i) {
        }
        restoreDomains(backup);
        // right branch x != v
        try {
            y.dom.remove(v);
            fixPoint();
            dfs(onSolution);
        } catch (Inconsistency i) {
        }
    }
}
```

Clone domains

Branch (left) and Fix-Point

Restore domains

Branch (right) and Fix-Point

Domain implementation: java.util.BitSet

```java
public class Domain {

    private BitSet values;

    public Domain(int n) {                private Domain(BitSet dom) {
        values = new BitSet(n);               this.values = dom;
        values.set(0, n);                 }
    }

    public boolean isFixed() { size() == 1; }
    public int size() { return values.cardinality(); }
    public int min() { return values.nextSetBit(0); }

    public boolean remove(int v) {
        if (0 <= v && v < values.length()) {
            if (values.get(v)) {
                values.clear(v);
                if (size() == 0) throw new TinyCSP.Inconsistency();
                return true;
            }
        }
        return false;
    }

    public void fix(int v) {
        if (!values.get(v)) throw new TinyCSP.Inconsistency();
        values.clear();
        values.set(v);
    }

    public Domain clone() {
        return new Domain((BitSet) values.clone());
    }
}
```

Performances

What to measure ?

- The number of nodes (recursive calls)
- The time

- Let’s compare the three approaches
  - NQueensChecker (generate and filter)
  - NQueensPrune (prune the search when violation detected on prefix of decisions)
  - NQueensTinyTSP (using the tiny CSP solver)

NQueensChecker

N	Nodes	Time (ms)	#solutions
8	19173961	167	92
9	435.848.050	4,526	352
10	11.111.111.111	101,497	724

NQueensPrune

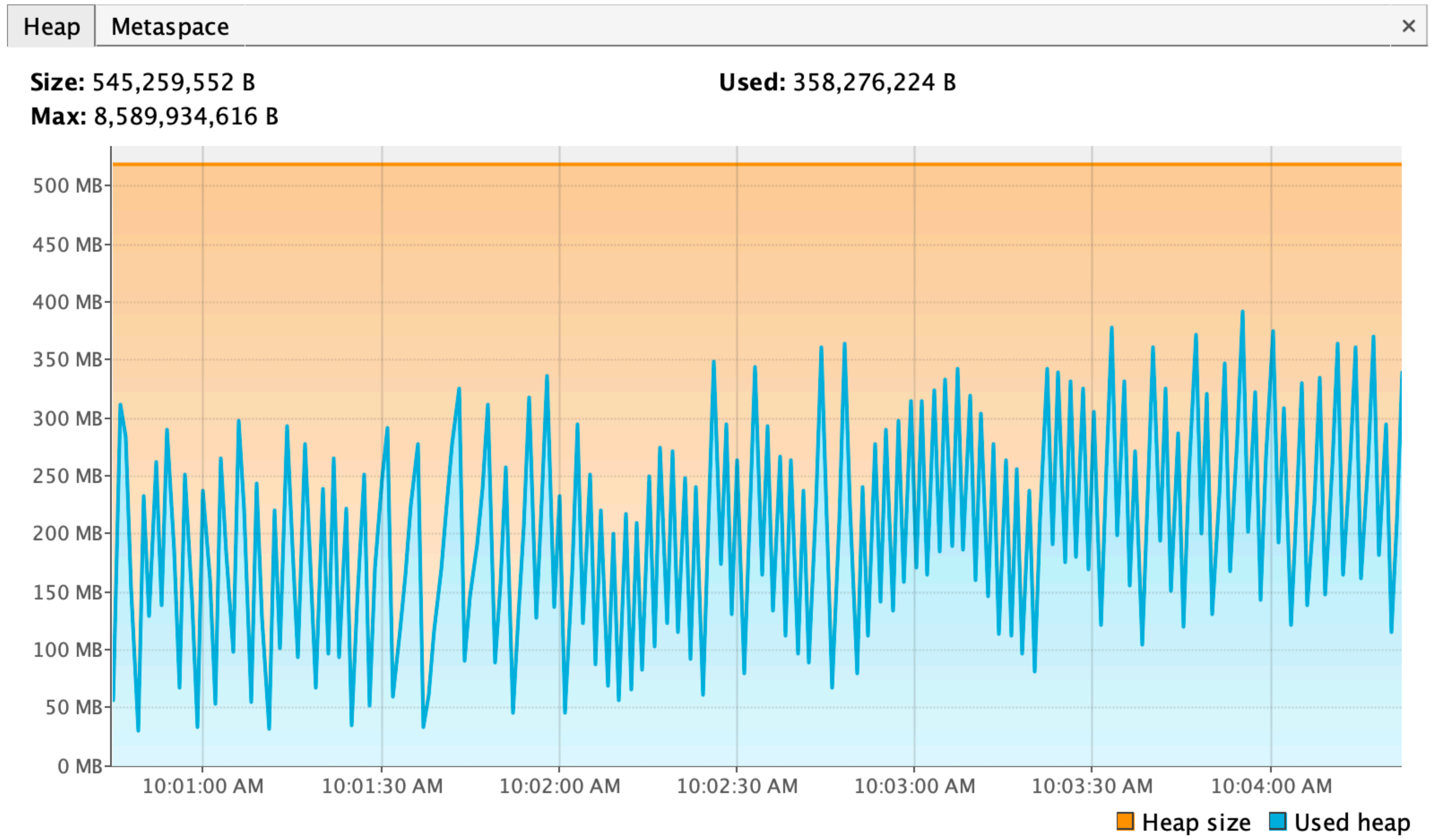
N	Nodes	Time (ms)	#solutions
12	856.189	130	14.200
13	4.674.890	690	73.712
14	27.358.553	4.550	365.596
15	171.129.072	30.138	2.279.184

NQueensTinyCSP

N	Nodes	Time (ms)	#solutions
12	102.531	2.439	14.200
13	73.712	11.999	73.712
14	2.934.559	72.753	365.596
15	17.543.706	477.324	2.279.184

Where do we loose time in NQueensTinyCSP ?

- Profiler (Visual VM https://visualvm.github.io)



Name	Total Time (CPU)		Total Time	
tinycsp.examples.NQueensTinyCSP.**main** ()	152,296 ms	(23.3%)	152,296 ms	(23.3%)
tinycsp.TinyCSP.**dfs** ()	152,296 ms	(23.3%)	152,296 ms	(23.3%)
tinycsp.TinyCSP.**fixPoint** ()	67,361 ms	(10.3%)	67,361 ms	(10.3%)
tinycsp.TinyCSP$Inconsistency.**<init>** ()	59,579 ms	(9.1%)	59,579 ms	(9.1%)
tinycsp.Domain.**fix** ()	53,956 ms	(8.2%)	53,956 ms	(8.2%)
tinycsp.NotEqual.**propagate** ()	48,920 ms	(7.5%)	48,920 ms	(7.5%)
tinycsp.Domain.**size** ()	41,484 ms	(6.3%)	41,484 ms	(6.3%)
tinycsp.Domain.**isFixed** ()	41,381 ms	(6.3%)	41,381 ms	(6.3%)
tinycsp.TinyCSP.**backupDomains** ()	16,493 ms	(2.5%)	16,493 ms	(2.5%)
tinycsp.TinyCSP.**restoreDomains** ()	9,710 ms	(1.5%)	9,710 ms	(1.5%)
tinycsp.Domain.**remove** ()	8,032 ms	(1.2%)	8,032 ms	(1.2%)
tinycsp.TinyCSP.**firstNotFixed** ()	1,591 ms	(0.2%)	1,591 ms	(0.2%)
tinycsp.TinyCSP.**lambda$firstNotFixed$0** ()	493 ms	(0.1%)	493 ms	(0.1%)
tinycsp.TinyCSP$$Lambda$17.0x0000000800c03458.**test** ()	493 ms	(0.1%)	493 ms	(0.1%)

One source of inefficiency: The Fixpoint Algorithm

```
fixPoint()
{
  repeat
    select a constraint c;
    if c is infeasible given the domain store then
      return failure;
    else
      apply the pruning algorithm associated with c;
  until (no constraint can remove any value);
  return success;
}
```

Data: The CSP $\langle X, \mathcal{D}^0, C \rangle$
Result: The greatest fixpoint domain
$pruningNeeded \leftarrow true$
$\mathcal{D} \leftarrow \mathcal{D}^0$
while $pruningNeeded$ **do**
    $\mathcal{D}^p \leftarrow \mathcal{F}_C(\mathcal{D})$
    $pruningNeeded \leftarrow \mathcal{D}^p \neq \mathcal{D}$
    $\mathcal{D} \leftarrow \mathcal{D}^p$
end

If no domain of a variable of the constraint c was changed since last time it was executed, is it worth executing it again?

Improved Fixpoint Algorithm: Data-Driven

- The first algorithm is "naïve":
  - It invokes $\mathscr{F}_c$ on every constraint c all the time.
- We can make this far better!

Data: a CSP $\langle X, D^0, C\rangle$
Result: the greatest fixpoint of the filtering algorithms for the constraints in C, starting from the domains D^0 of the variables of X

$Q \leftarrow C$
$D \leftarrow D^0$
while $|Q| > 0$ **do**
  $c \leftarrow$ **dequeue**(Q)
  $D' \leftarrow \mathcal{F}_c(D)$
  $V \leftarrow \{x \in \text{Vars}(c) : D'(x) \neq D(x)\}$
  if $|V| > 0$ **then**
    enqueue$(Q, \{c' \in C : |\text{Vars}(c') \cap V| > 0\})$
  $D \leftarrow D'$

Only enqueue the constraints with some domain change in their scope (including c itself)!

In next part, design an efficient CP solver

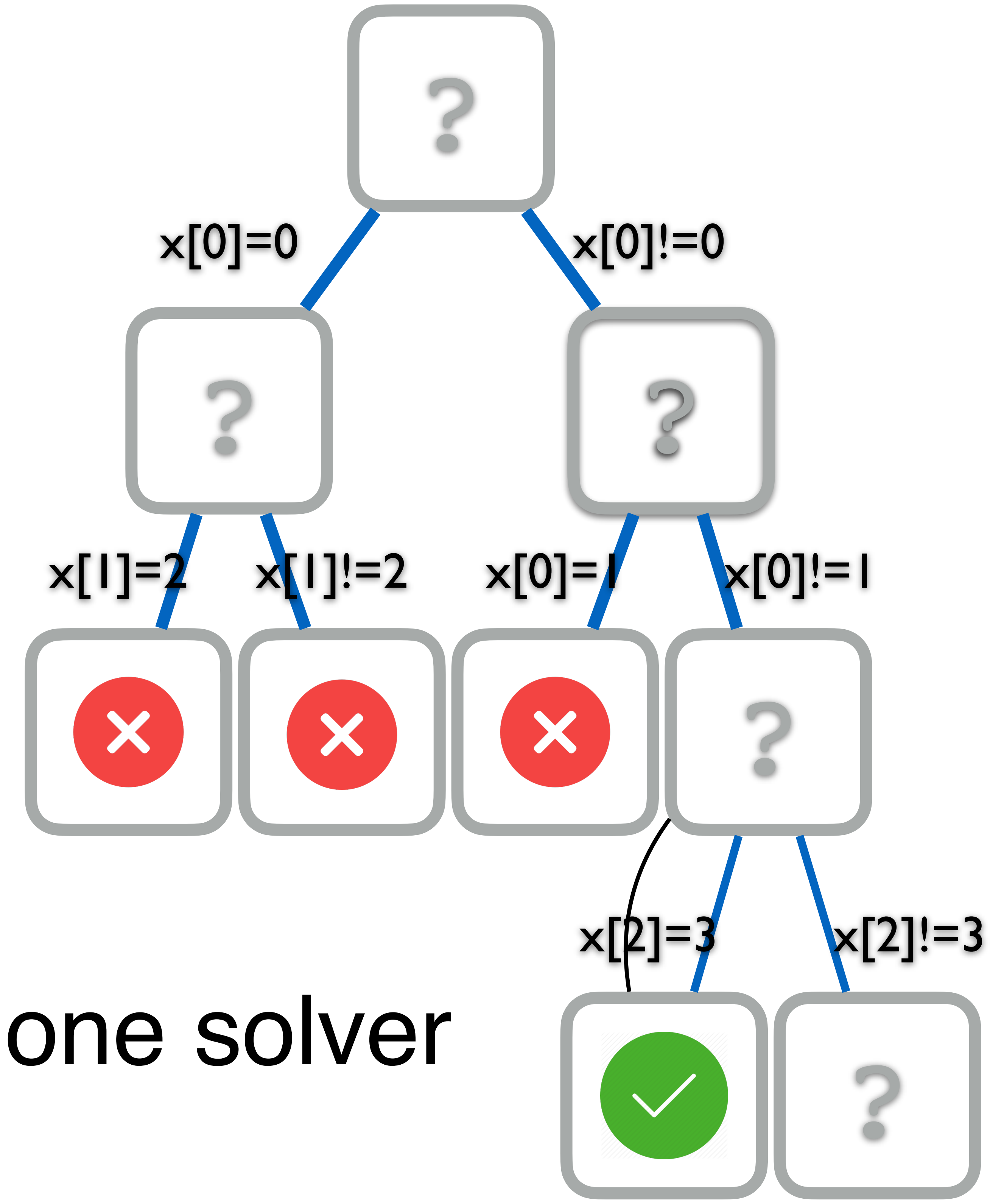
1. More fined grained mechanism for the fix-point and constraint propagation
2. Avoid creating “clones” of the domains and use memory efficient data-structure to restore domains without creating objects
3. Implement a generic and flexible search that can easily be used for complex branching decisions and complex heuristics

CP and Declarative Programming

Computational Paradigm of CP

- Complete method, not a heuristic, because a search-tree exploration:
  - Given enough time, it will find a / all solution(s) to a satisfaction problem.
  - Given enough time, it will find an optimal solution to an optimization problem.

- Focus on feasibility:
  - How to use constraints to prune the search space by removing domain values that cannot belong to any solution?



- Focus on reusability:
  - Can model many different problems with just one solver

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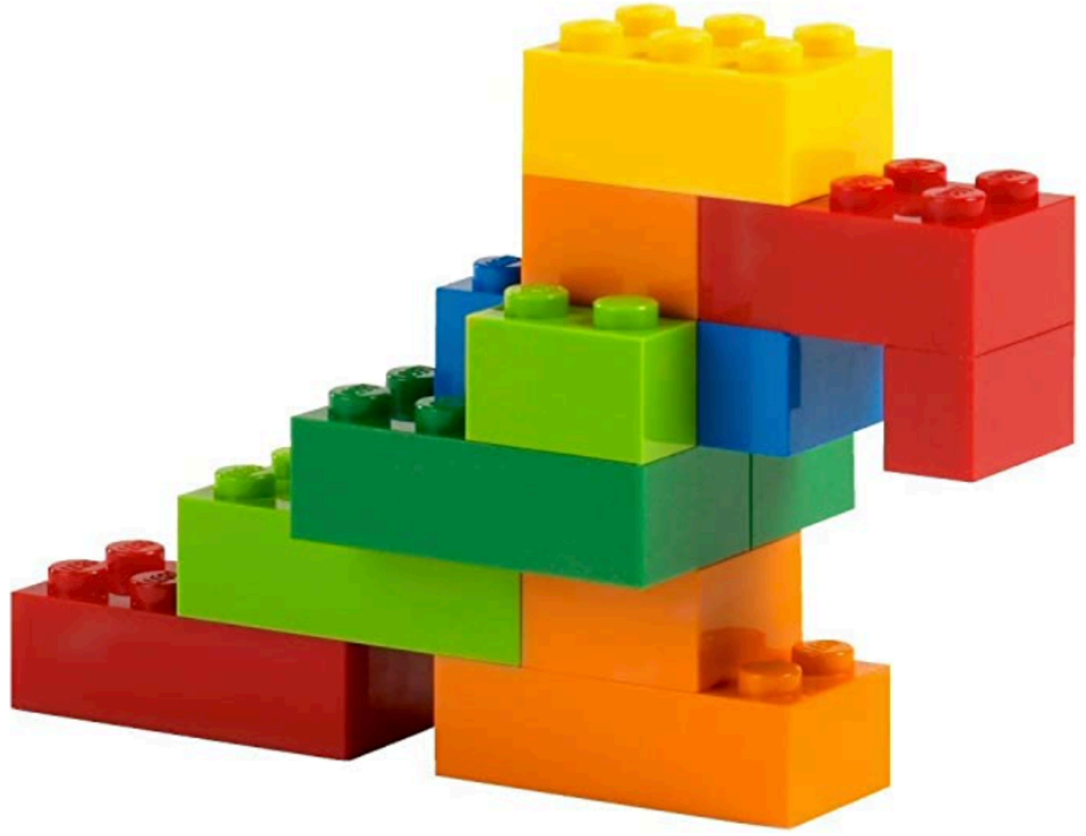
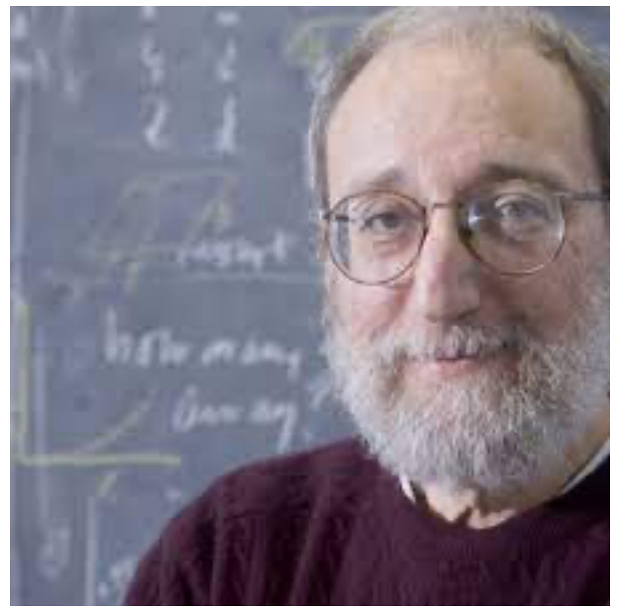
  - Can model many different problems with just one solver



Constraint Programming (CP)

"Constraint programming represents one of the closest approaches computer science has yet made to the Holy Grail of programming: the user states the problem, the computer solves it." (E. Freuder)

States, you mean like this?



Not yet … rather like this:

```
range R = 1..8;
var{int} q[R] in R;
solve {
   forall(i in R, j in R: i < j) {
      q[i] ≠  q[j];
      q[i] ≠  q[j] + (j - i);
      q[i] ≠  q[j] - (j - i);
   }
}
```

but who knows in the future ;-)

State Problem = Declarative Programming

Declarative programming is a *programming paradigm* that expresses the logic of a computation without describing its control flow.

Declarative programming for solving constrained combinatorial (optimization) problems means that you express the properties of solutions that must be found by “the solver”.

CP Slogan

CP = Model (+ Search)

Model description:

user API for
declarative programming

The algorithmic part:

finding a solution that
satisfies all the constraints, etc,

usually by exploring a search tree

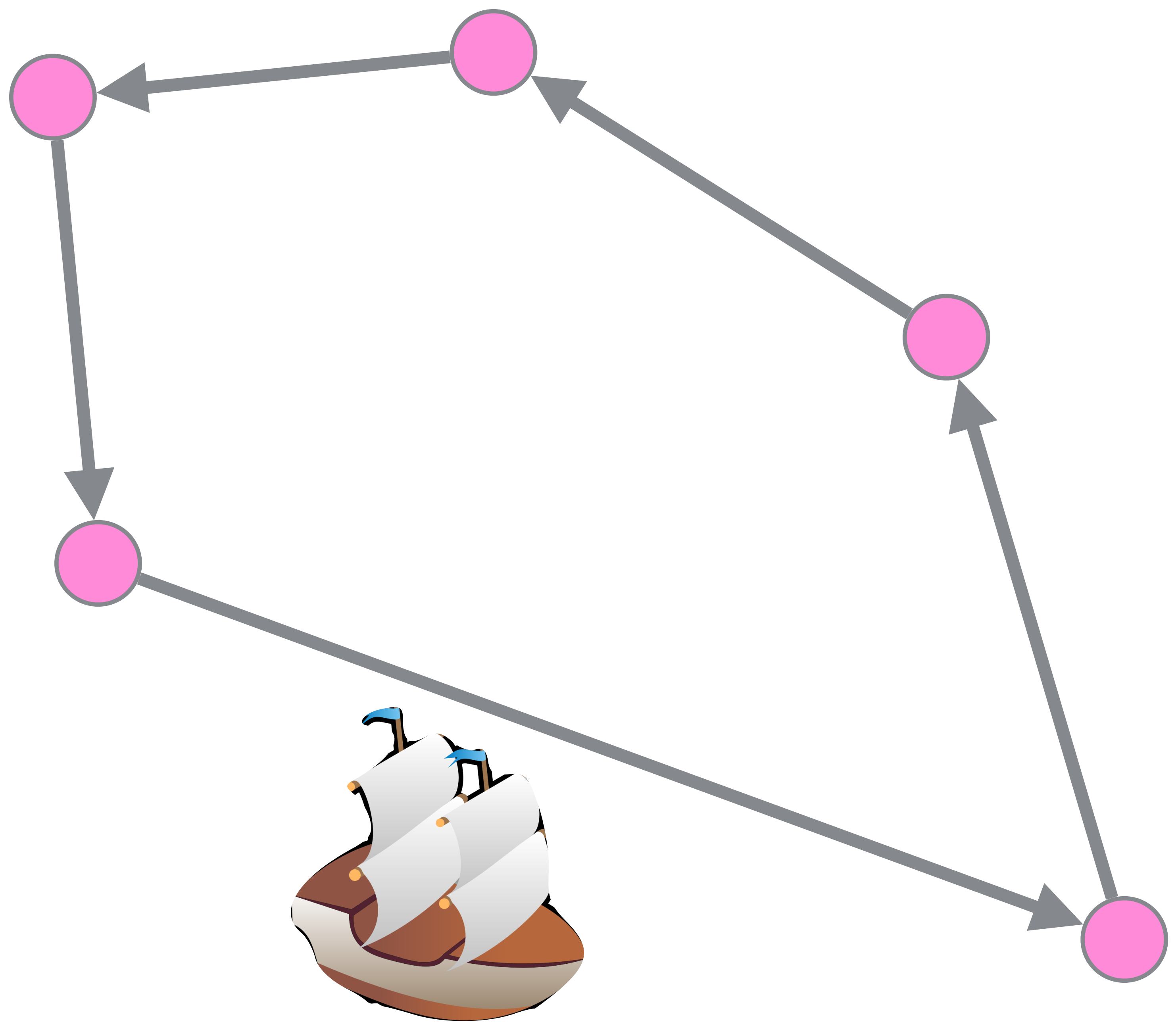
Model

A *model* of a constraint satisfaction problem has:

- Variables with sets of possible values, called *domains*:
  - Generally integer sets or integer intervals, such as $x \in \{5,9,10\}$, but also on floats, graphs, etc.
- Constraints on the variables:
  - Arithmetic ex: $3x + 10y = z$ (linear constraints are a special case!)
  - Logical ex: $x < y$ or $x > z$ (predicate logic)
  - Combinatorial ex: Circuit($x_1,\ldots,x_n$) (structural requirements)

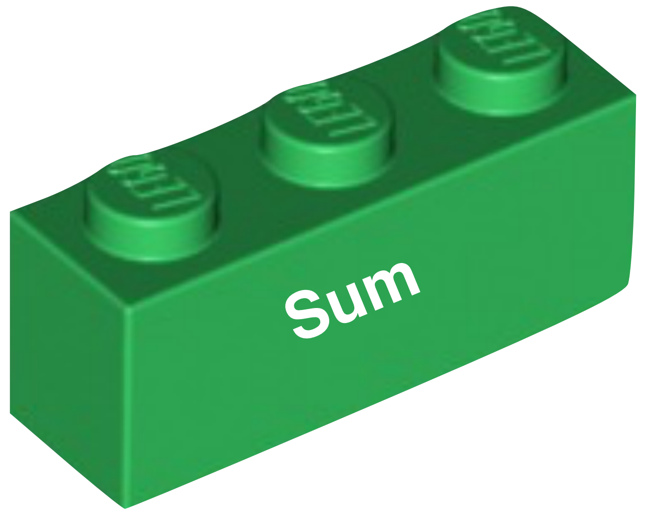
Variables: Example

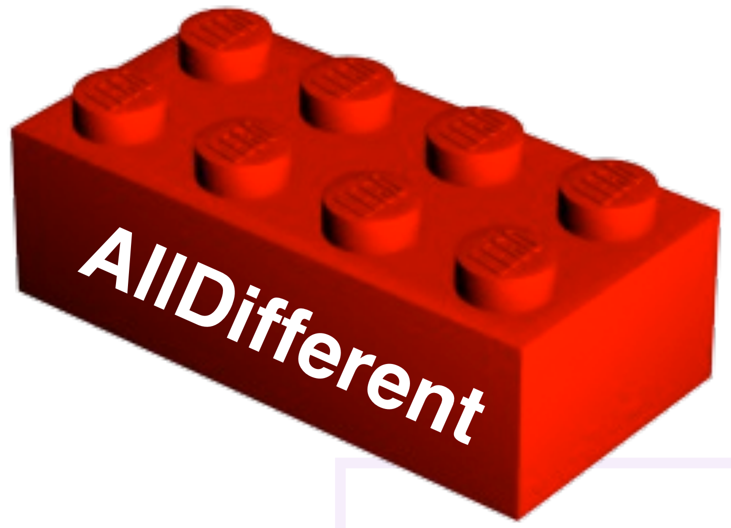
- *Variable* = a decision that should be made.
- *Domain* = finite set of possible values for the variable.
- Example:
  - x_i = the city to visit after city i in a tour for the traveling salesperson (TSP);
  - $D(x_i) = \{0, 1, \ldots, i-1, i+1, \ldots, n-1\}$, where n = #cities: all the possible values for x_i.



Constraints: Examples

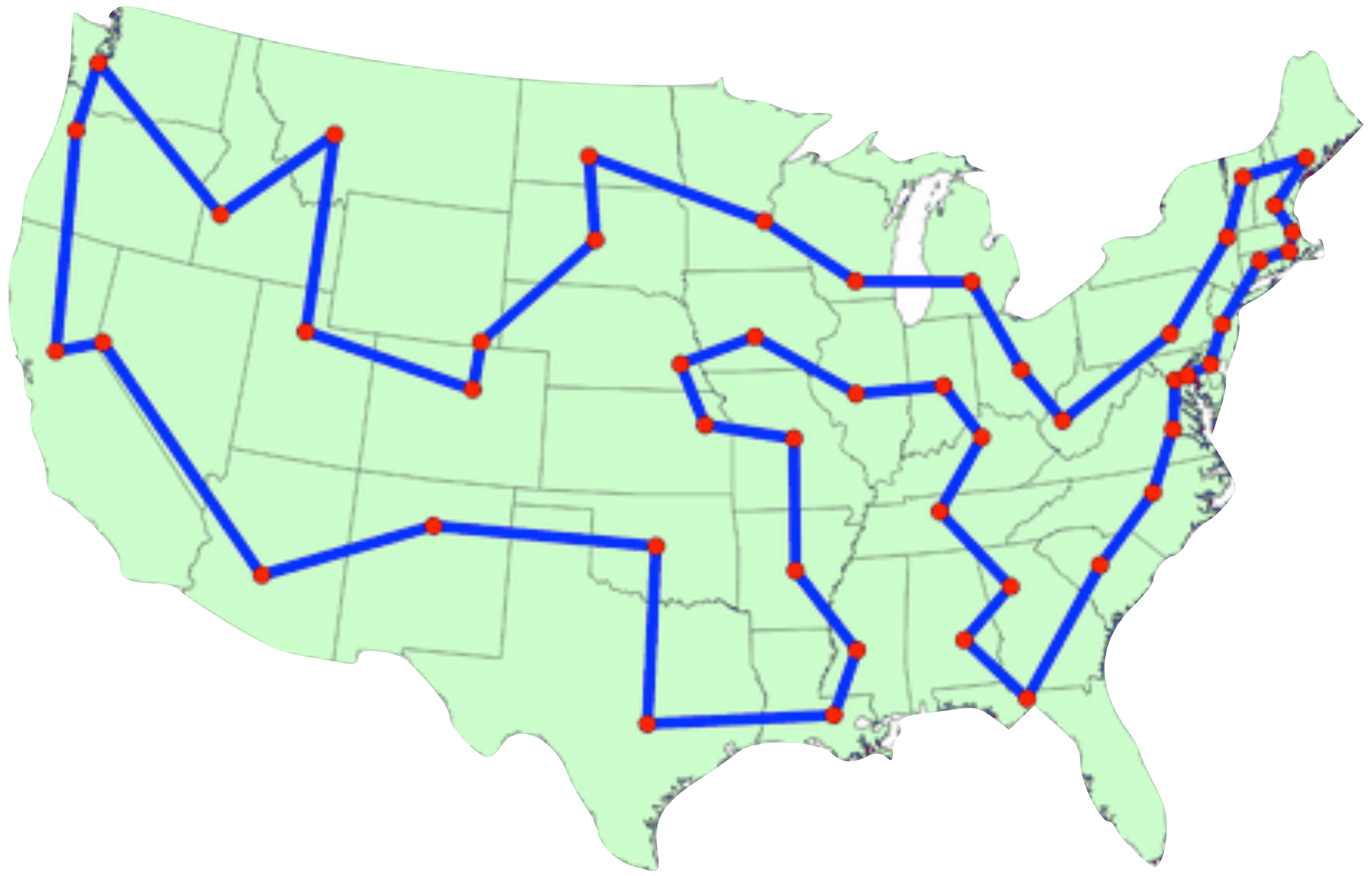
Arithmetic	Logical	Combinatorial
$Sum(x[], y) \equiv \left(\sum_i x_i \right) = y$	$y_i = c \Leftrightarrow y_{ic} = 1$	$AllDifferent(x[])$



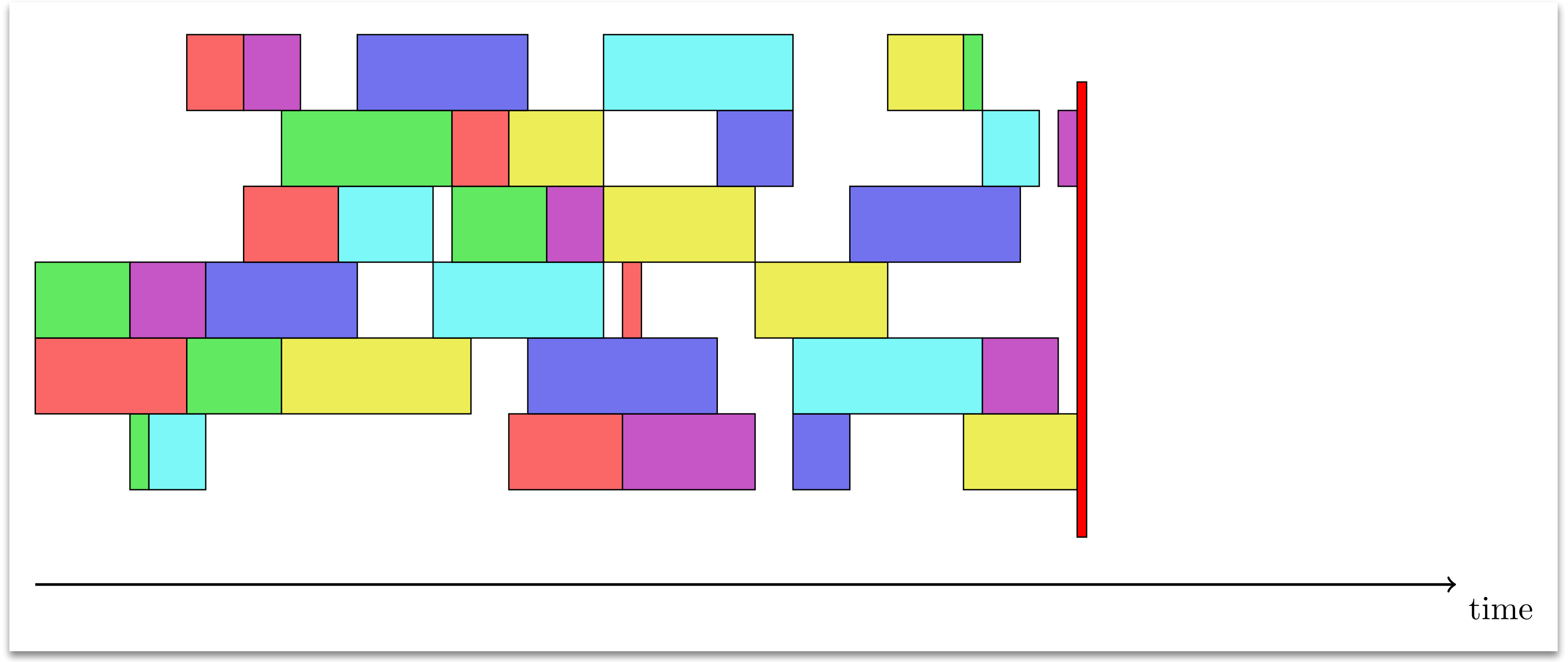


Application Domains

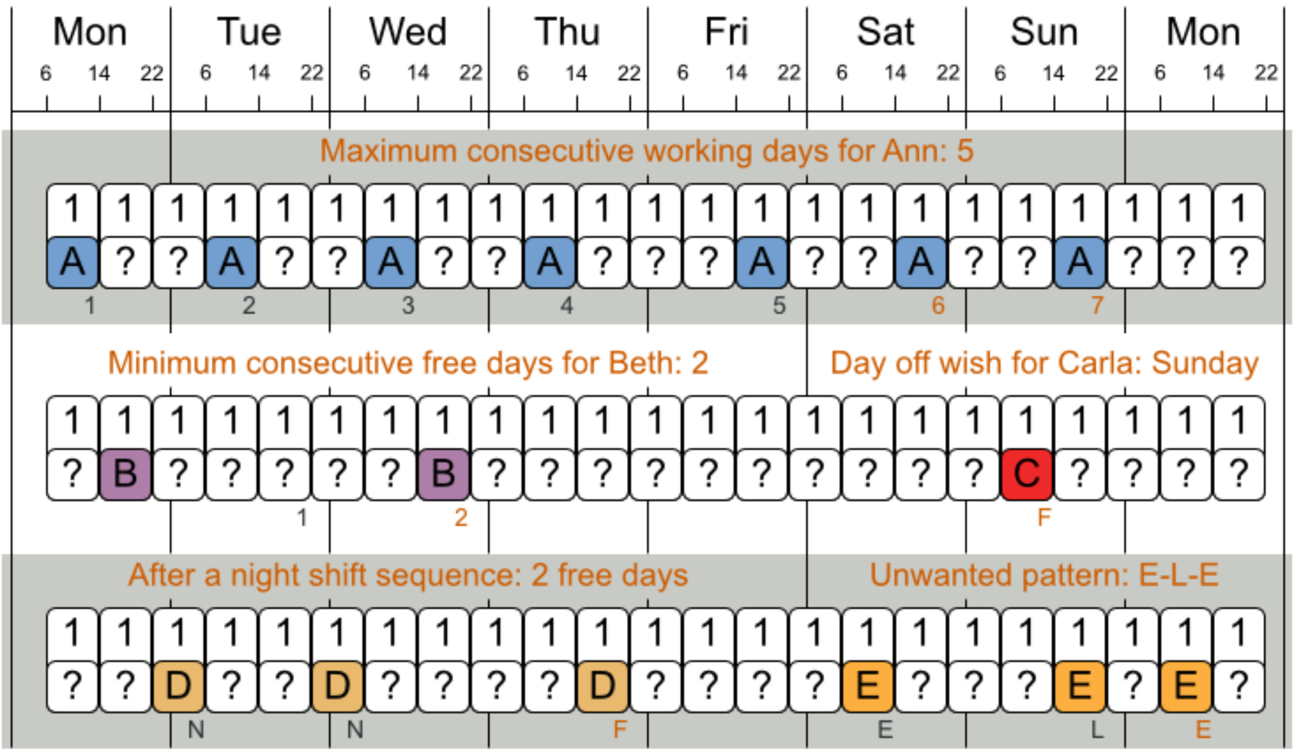
Routing



Scheduling

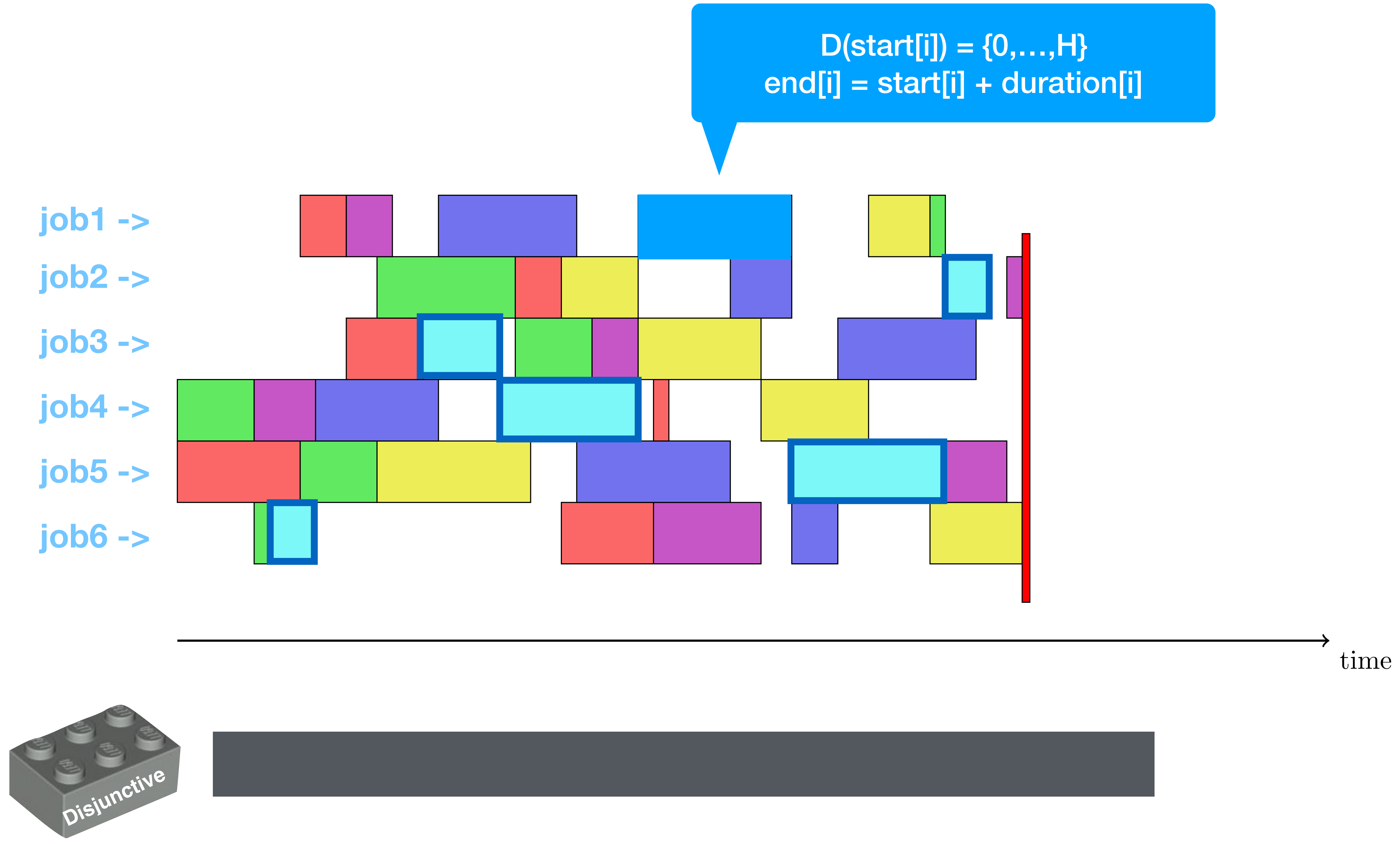


Rostering



A Combinatorial Constraint for Jobshop?

- Yes!
  - Disjunctive(…)



TSP Modeling: CP vs MIP

MIP

$$\text{minimize} \sum_{i,j} d_{ij} \cdot x_{ij}$$

$$\text{subject to} \sum_{i \in V} x_{ij} = 2 \quad \forall i \in V$$

$$\sum_{i,j \in S, i \neq j} x_{ij} \leq |S| - 1 \quad \forall S \subset V, S \neq \phi$$

$$x_{ij} \in \{0,1\}$$

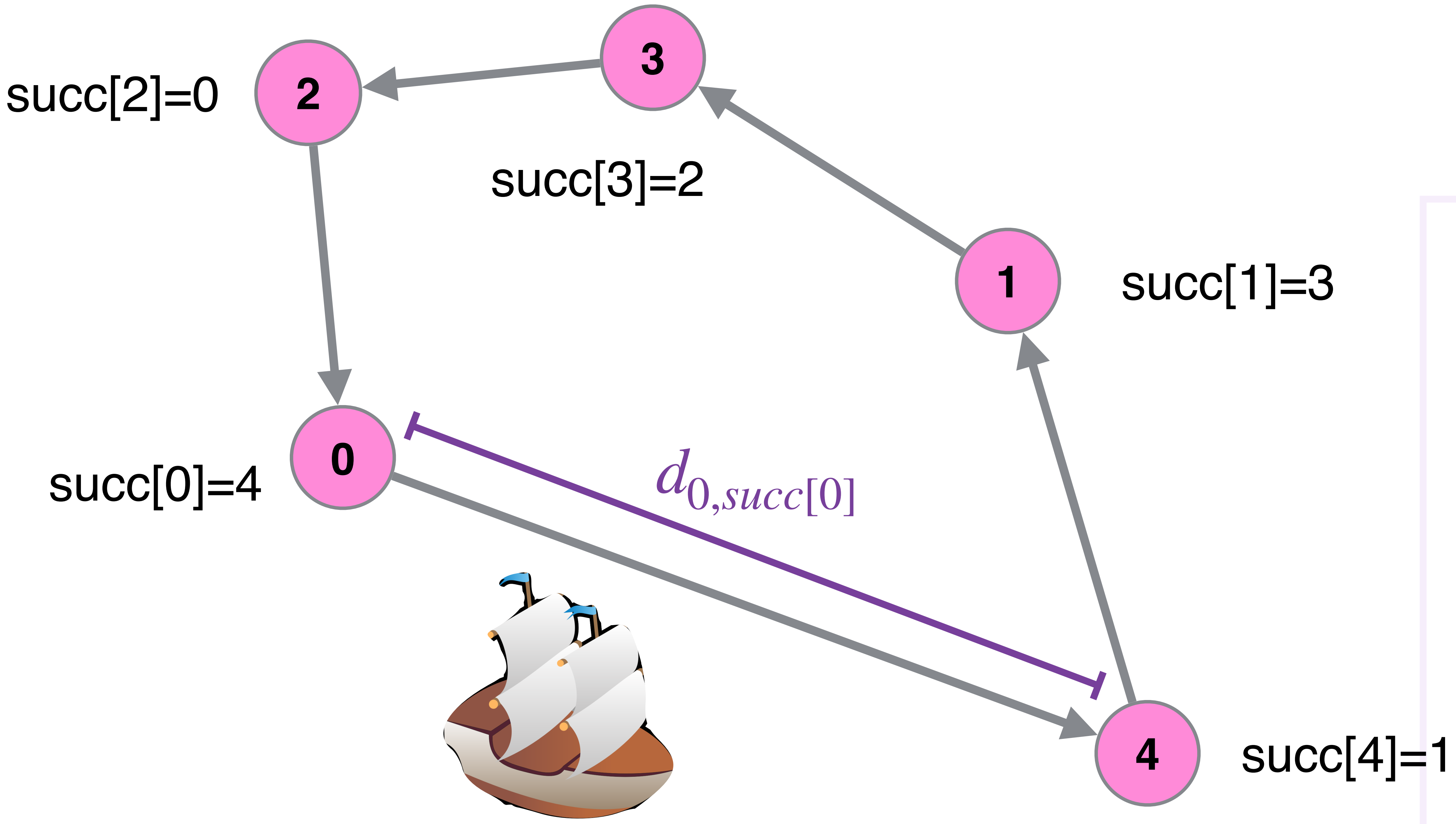
CP

index an array with variables!

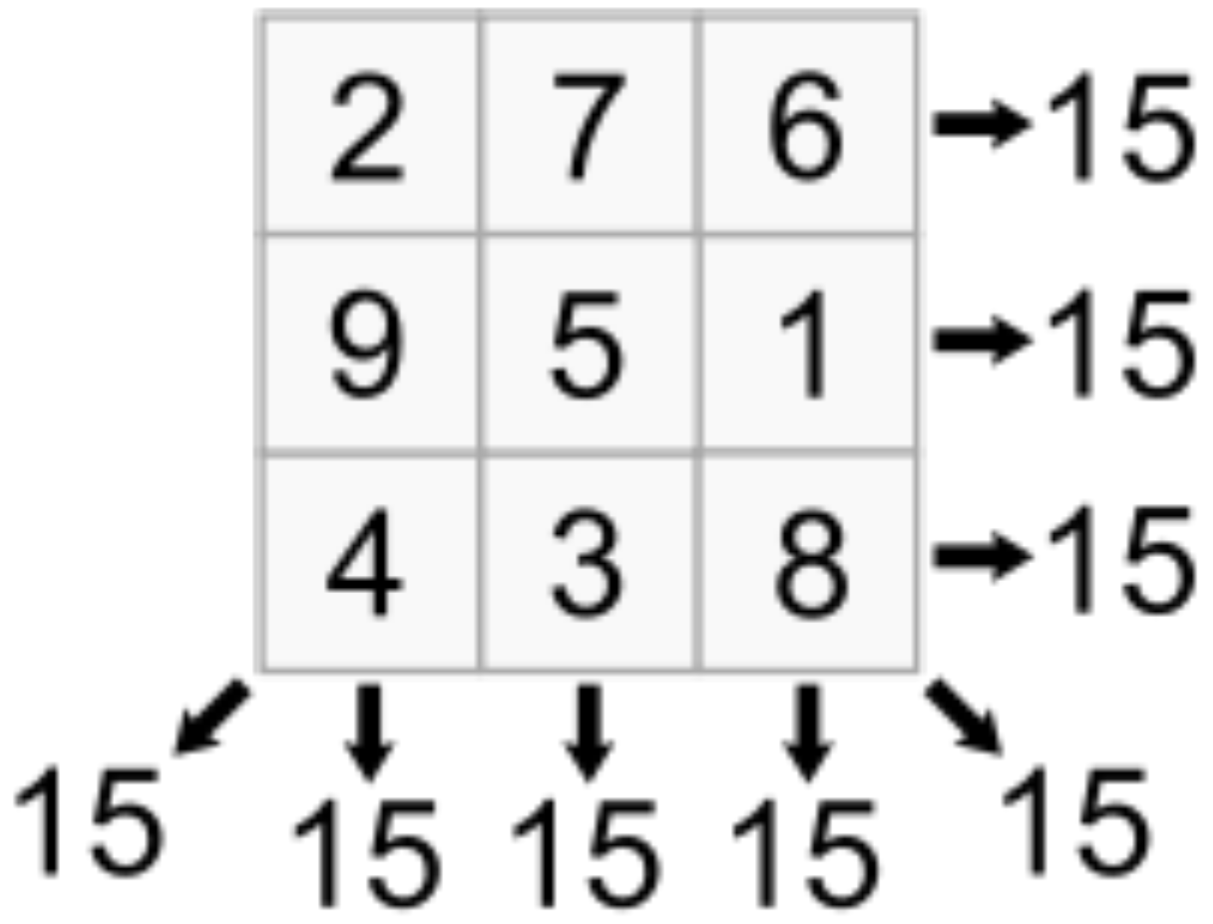
$$\text{minimize} \sum_{i \in V} \boxed{d_{i,succ[i]}}$$

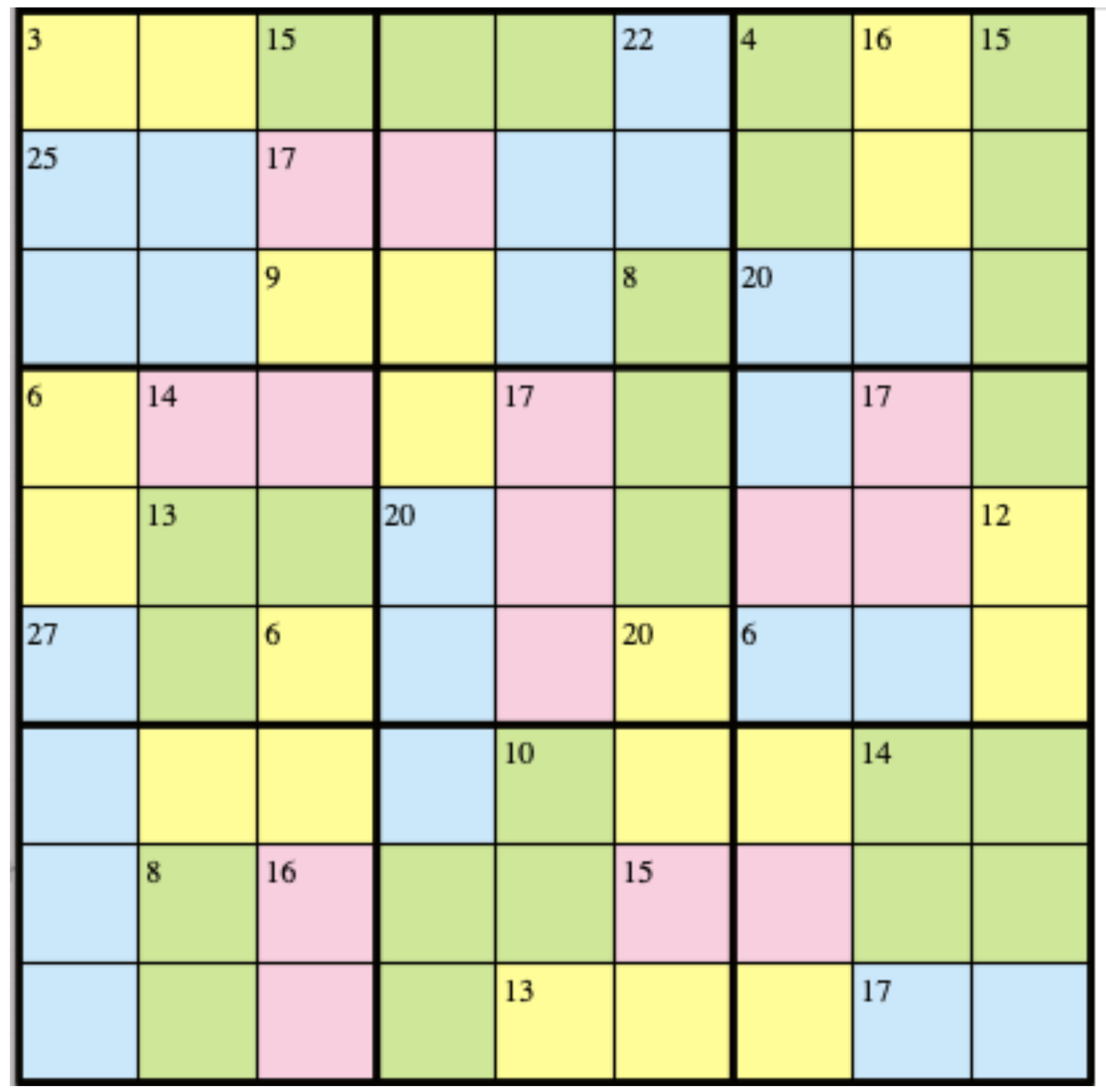
$$\text{subject to Circuit}(succ)$$

$$succ[i] \in \{0,\ldots,i-1, i+1, n-1\}$$



Projects Magic Square + Killer Sudoku





Sum Constraint + Less Or Equal Update

Symmetries

Killer Sudoku