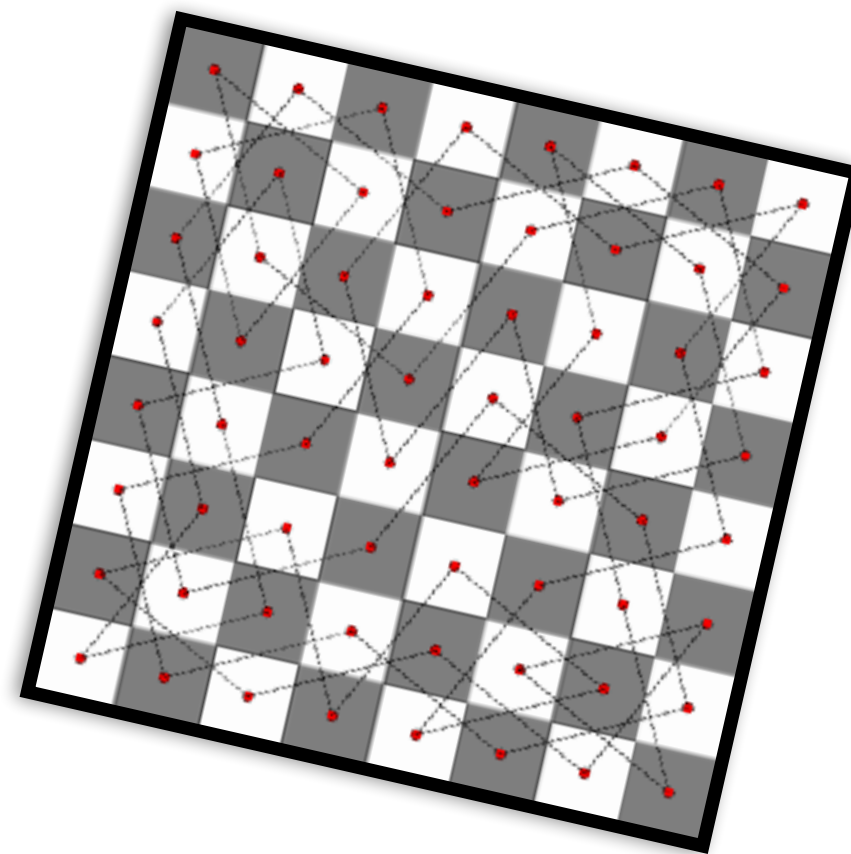


# Advanced Algorithms for Optimization

LINFO2266

## Introduction

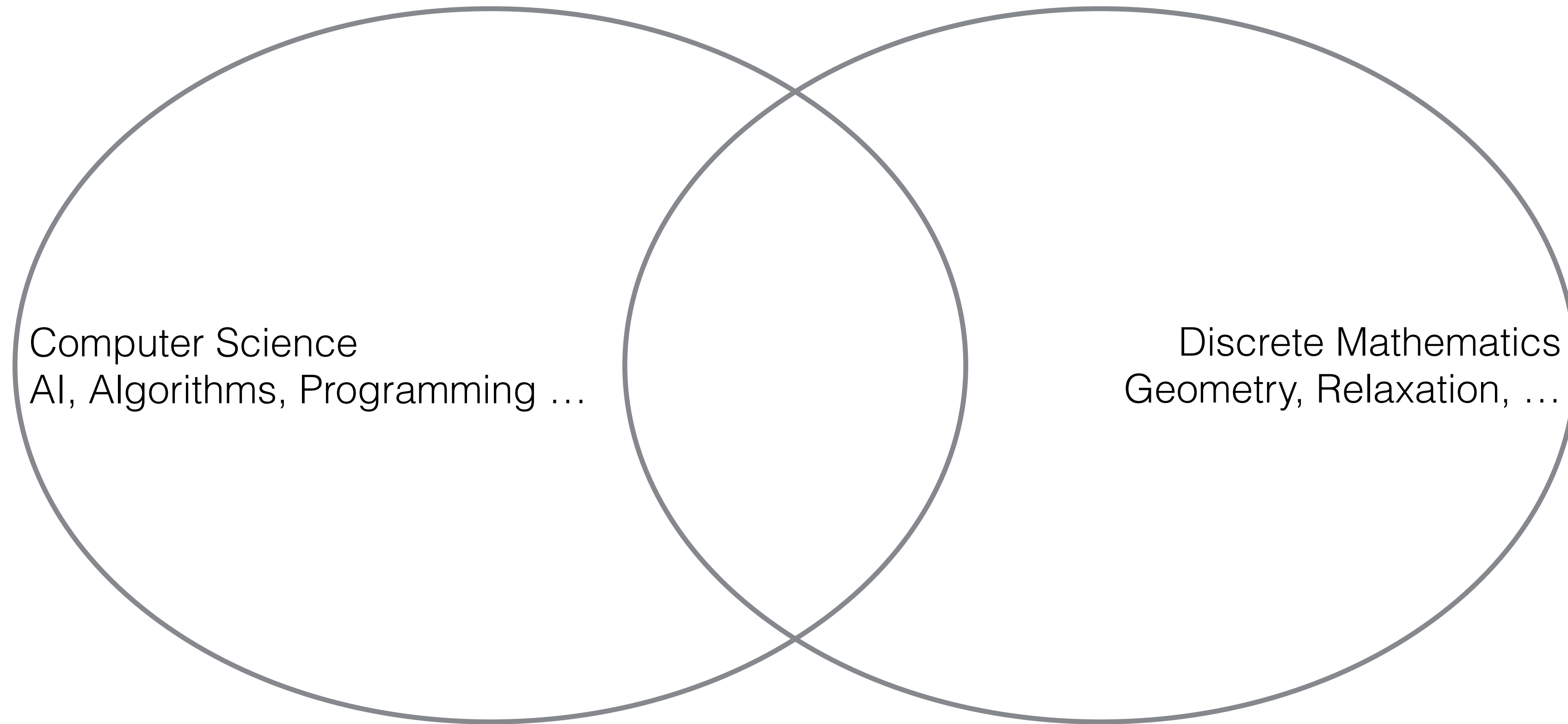
Prof: Pierre Schaus  
TAs: Alice Burlats, Amaury Guichard



# Optimization is everywhere!

- Logistics (rail, containers, vehicle routing, warehouse)
- Human Resource (nurse rostering, course, etc)
- Scheduling (industry production, tournaments)
- Networking (TE, routing, network design)
- Data-science and Machine Learning (Neural Networks, etc)
- ...

# A multi-disciplinary field



Since this course is more CS oriented, we will focus more on algorithms and computational aspects



# Why study Algorithms for Optimization

- Many large companies (IBM, Google, N-SIDE, Artelis, Informs, Quintiq, Dassault, PSI, OM-Partners, Infrabel, Amazon, etc)
  - The return on investment (ROI) when deploying optimization can be huge, especially when large volumes.
- Because implementing advanced algorithms, able to solve problems that you cannot even solve manually is fun!
- Because whatever you do later, being « algorithm and optimization » minded can be useful and a real asset.

# Objective of the course

- Be able to understand and formalize a constrained optimization problem.
- Be able to solve and implement well known discrete optimization algorithms using a well designed and generic approach when possible.
- Be able to understand the advantage and weaknesses of each approaches.

# Pedagogy = Project based Learning

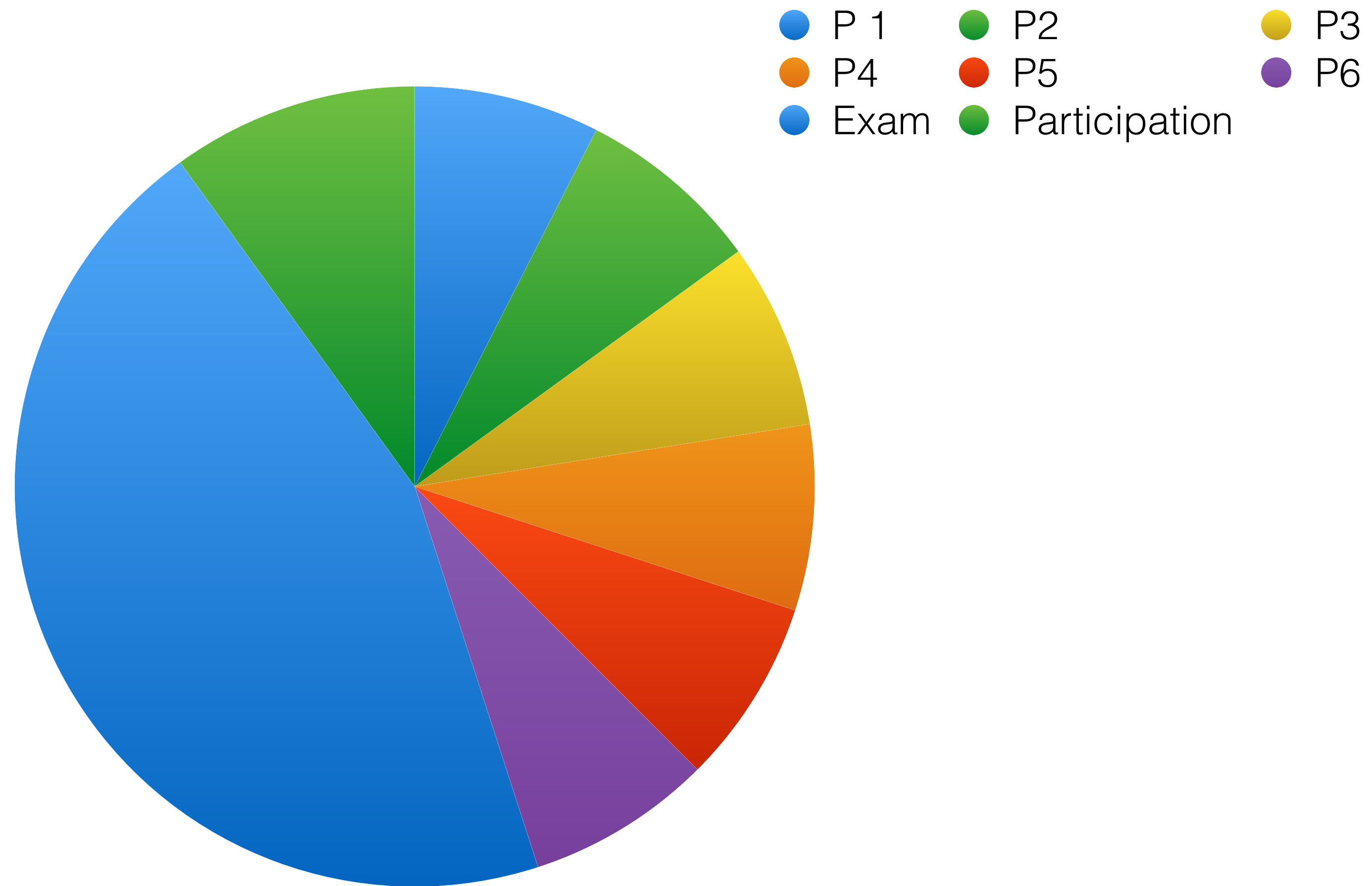
- *“I hear and I forget. I see and I remember. I do and I understand”*
- 6 Projects = 45% of the final grades (!!!! no second chance in August)
  - Individual Projects
  - A lot to implement (complex code), tested on ingenious
  - A report to write submitted on gradescope and evaluated manually (pdf)



- Exam = 45% of the final grade (second chance in August)
- Participation = 10% (no second chance in August)



# Grading



**Small projects cannot be done again in august** , only one chance

# Optimization Contest

- Introduced next week



Depending on your ranking, they can earn 0, 1, 2, 3, 4, or 5 points.

If you earn  $X$  points, and your grade on the projects is  $Y$  (on 20),

Your final grade is  $\max \left( Y, \frac{Y}{20} \times (20 - X) + X \right)$



# Individual Projects ⚠

It means that any source code of a project estimated to be

- copied or inspired by the one of another student, or
- copied or inspired by a source code found on the internet or another source,

Will result in a **zero grade for the student at all the projects.**

The same consequences will hold for a student that voluntarily shares his code or make available to other students.



# What is allowed

- Students are encouraged to exchange insightful ideas and observations.
- But they should never share code for solving the exercises.
- Your GitHub repository must remain closed.

# Lectures Agenda and deadlines for the projects

| Week | Wed. 4h15PM | Lecture                      | Start Project              | Deadline Project (14h)     |
|------|-------------|------------------------------|----------------------------|----------------------------|
| 1    | 17/09/2025  | Intro + DP                   | P1: DP                     |                            |
| 2    | 24/09/2025  | Benchmarking + Contest Intro | Competition                |                            |
| 3    | 01/10/2025  | Branch and Bound             | P2: Brand and Bound        | P1: DP                     |
| 4    | 08/10/2025  | Lagrangian Relaxation        |                            |                            |
| 5    | 15/10/2025  | Linear Programming           | P3: LP and Flows           | P2: Branch and Bound       |
| 6    | 22/10/2025  | Network Flows                |                            |                            |
| 7    | 29/10/2025  | Smart-Week                   |                            | P3: LP and Flows           |
| 8    | 05/11/2025  | Local Search                 | P4: Local Search           |                            |
| 9    | 12/11/2025  | Contest Consultancy          |                            |                            |
| 10   | 19/11/2025  | Constraint Programming       | P5: Constraint Programming | P4: Local Search           |
| 11   | 26/11/2025  | Contest Consultancy          |                            |                            |
| 12   | 03/12/2025  | A-star                       | P6: A-star                 | P5: Constraint Programming |
| 13   | 10/12/2025  | Contest Consultancy          |                            |                            |
| 14   | 17/12/2025  |                              |                            | P6: A-star                 |
| 15   | 19/12/2025  |                              |                            | Contest                    |



# Generative AI: What is allowed

Can be used but **academic honesty** and originality remain paramount.

- Original Work: Your submission should be primarily your own work. Directly copying and pasting solutions from AI outputs without understanding or modification is discouraged.
- Source Indication: Whenever you use generative AI to assist in your assignment, you are required to indicate so by providing a brief comment in your code on how the AI was used. For example:

*# Used AI to suggest optimization for this loop.*

**for** i **in** range(10): ...

Failure to adhere to these guidelines => academic penalties.

**We may always request an interview about the projects!**

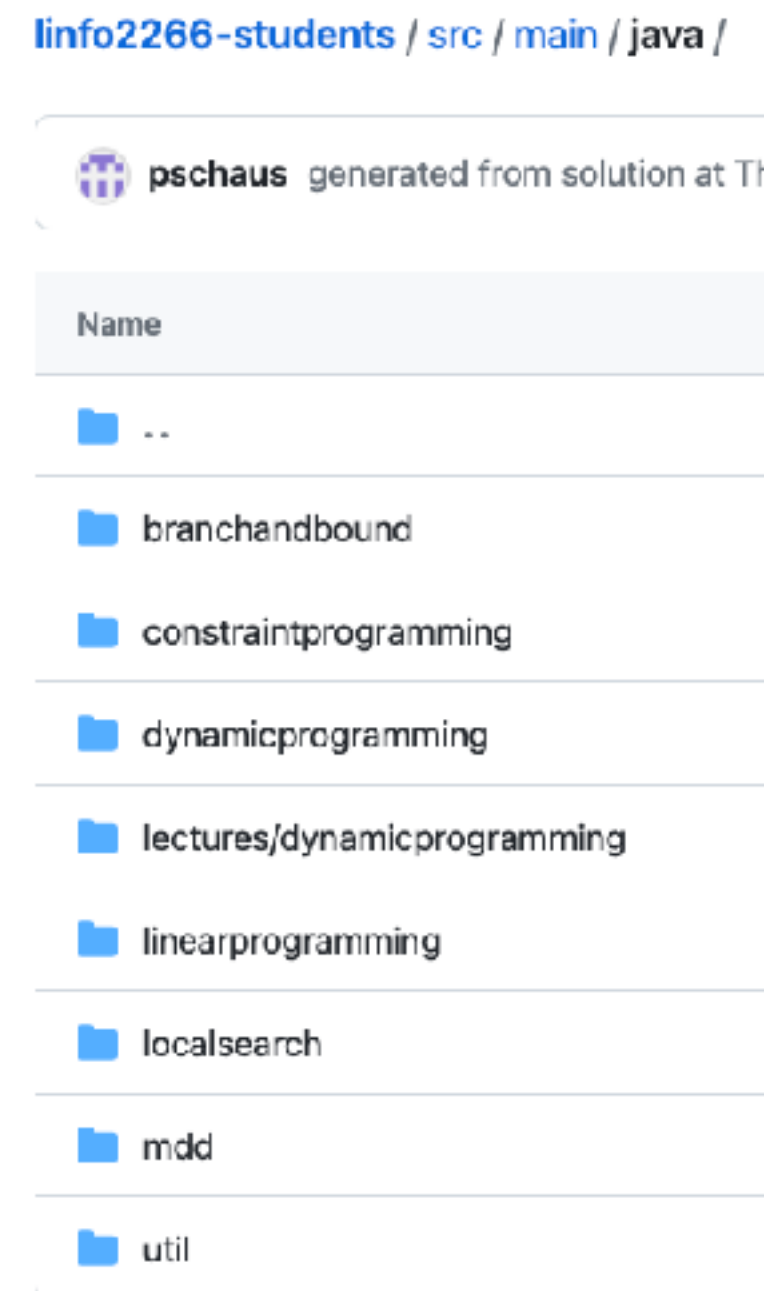
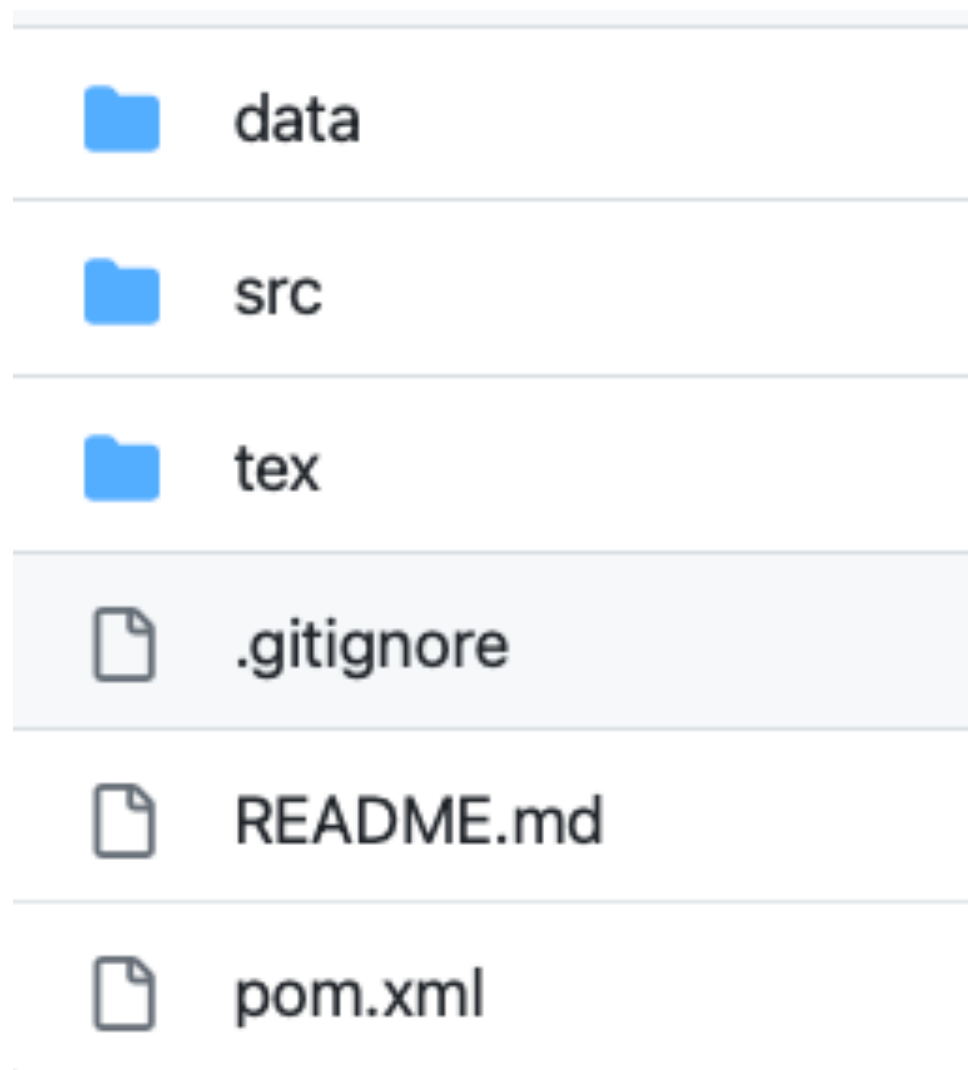


# The repository of the course

- Website with slides and project statements
  - <https://linfo2266.readthedocs.io>
- Inginious:
  - <https://inginius.info.ucl.ac.be/course/INGI2266>
- Github with the projects and latex templates
  - <https://github.com/pschaus/linfo2266-students>
- Gradescope for report submissions
  - <https://www.gradescope.com/courses/869042>

# Submission and projects, how it works ? 1/4

- Step 1: read and approve the “Honor Code Agreement Contract”.
- Step 2: Create your repository, you simply give your GitHub id and you are then invited to be the collaborator of a project with some source-code.



# Submission and projects, how it works ? 2/4

- Step 3: You code your solution to the current project (commit frequently to your repository)
- Step 4: Press “submit” button on the task of the current project

Code exercises - Dynamic Programming

Implement the functions from the [dynamicprogramming](#) package to pass the tests

Submit



# Submission and projects, how it works ? 3/4

- Inginius will then:
  - Pull your code (with your solution)
  - Run tests (some of them are hidden)
  - Grade your solution
- If happy with the grading, stop there, otherwise work harder.
- Not yet finished: The pdf report to submit on <https://www.gradescope.com/courses/869042> Entry code: 7E5W86



# Submission and projects, how it works ? 4/4

- Small questions (experiment, theory, etc). Answer (Ipad+Scan manually written are ok) or edit the PDF or do it from the latex source. Then upload the PDF.

|                   |  |
|-------------------|--|
| First & last name |  |
| NOMA UCLouvain    |  |

**LINFO2266: Advanced Algorithms for Optimization**

Project 1: Dynamic Programming

**Exercise 1     Modeling the Traveling Salesman Problem**

The Traveling Salesman Problem is a famous combinatorial optimization problem: where a salesman has to find the shortest tour visiting all cities from a given set  $N = \{1, \dots, n\}$ . The salesman can travel between every pair of cities  $i, j \in N$ , which are separated by a distance  $d_{ij}$ . Find a dynamic programming model for the Traveling Salesman Problem.

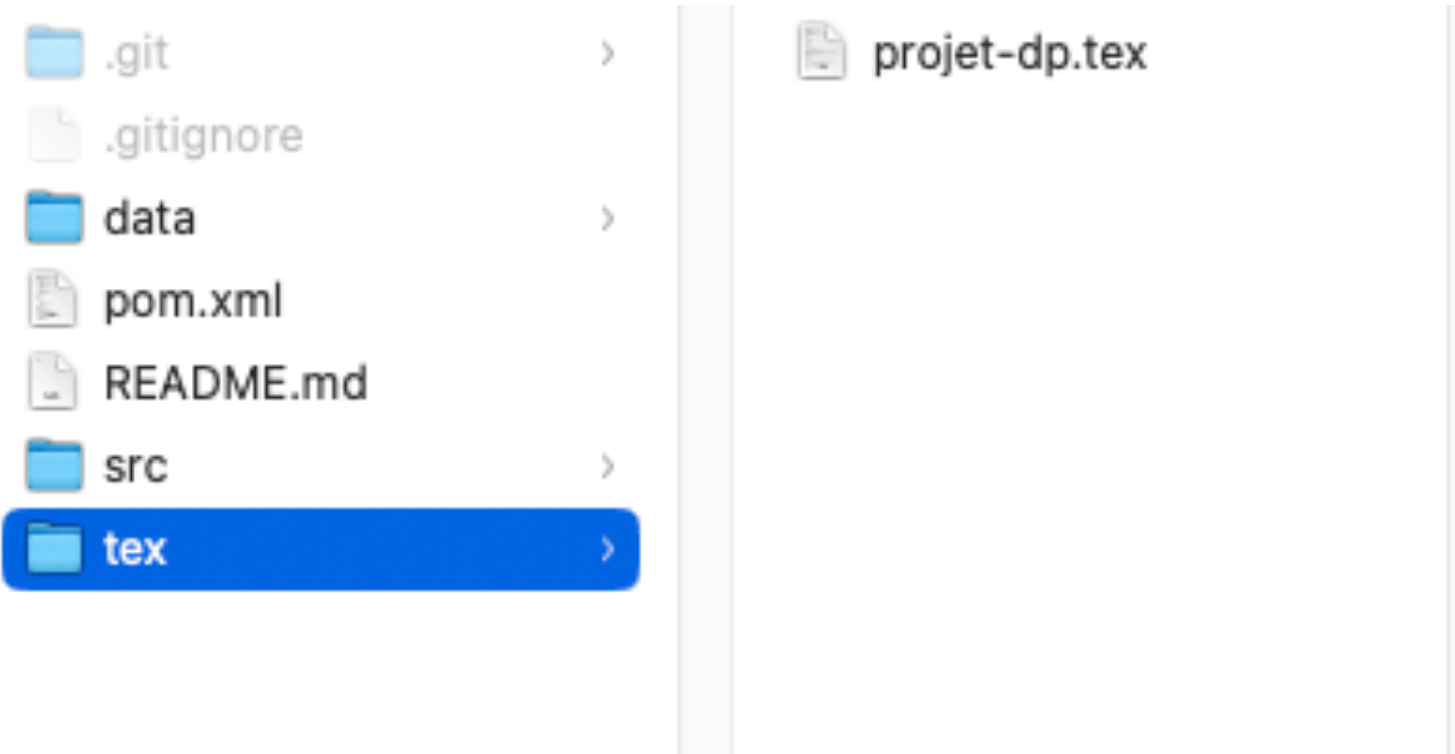
1. What information is contained in the states of the model?

2. Give the Bellman recurrence equation that computes the optimal value for each state of the model. You can assume that the salesman starts at city 1.

3. Give the base case(s) of the recurrence, and the case that gives the optimal solution of the problem.

4. What is the space complexity of a program computing the solution of your dynamic programming model, with respect to the number of cities  $n$ . Why?

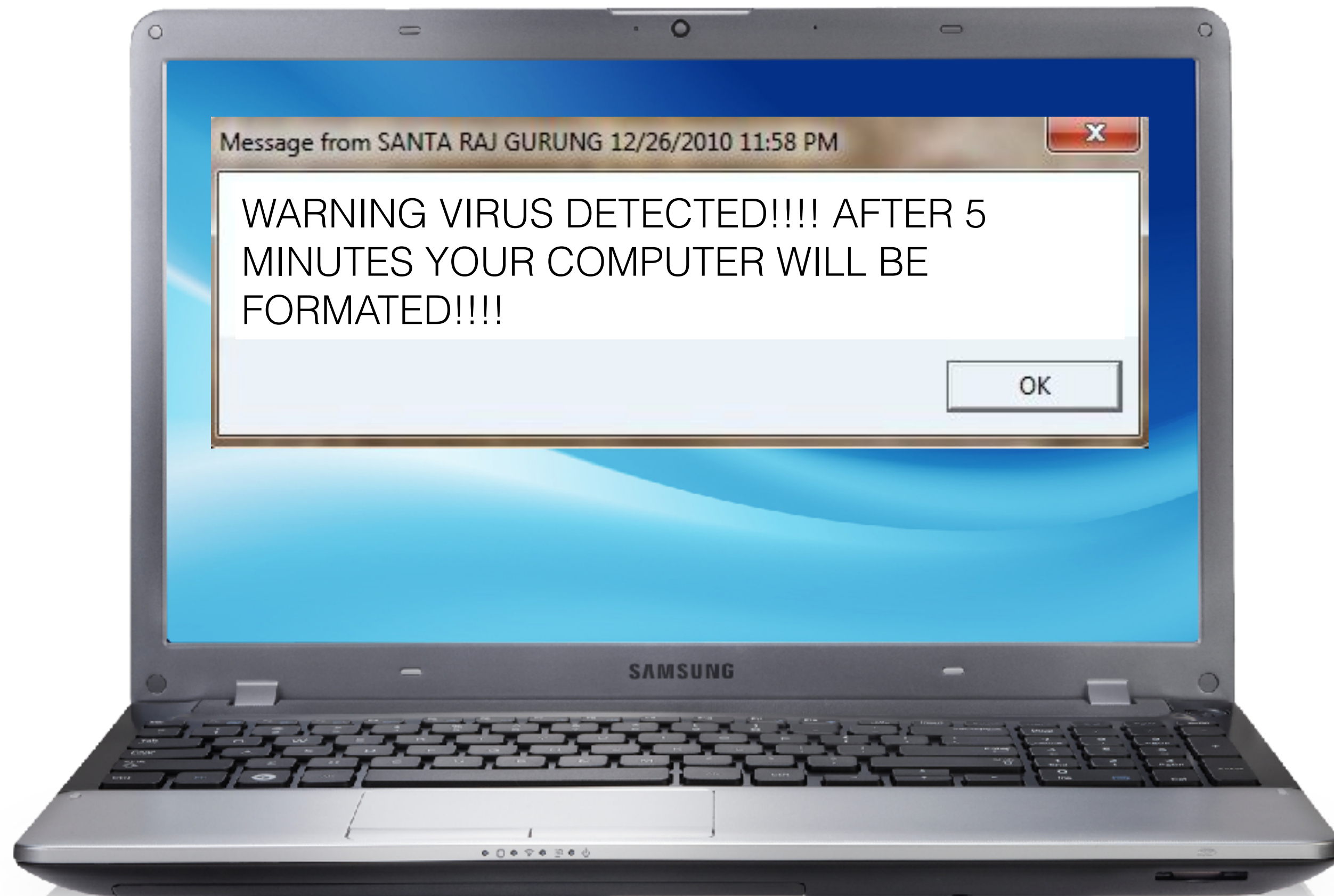
5. What about the time complexity? Explain why.



# Overview of some optimization techniques

- Motivating example
- In order to show you how I generally decide what approach I should use to solve an optimization problem

# Motivation



10 GB

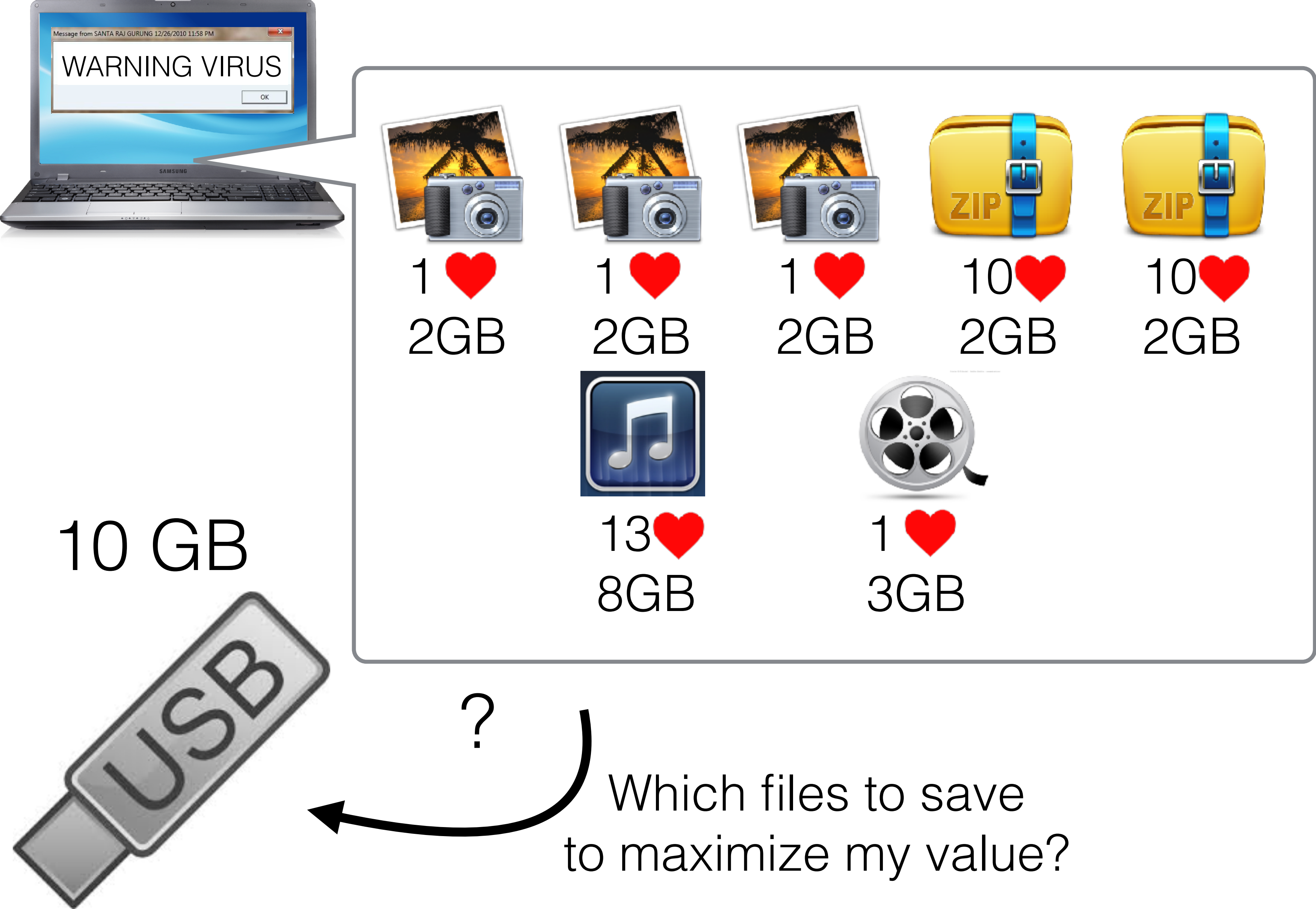


GB's of personal data  
lost !!!





# The problem



# Knapsack Problem

- the set of items:  $I$
- is item  $i$  selected:  $x_i \in \{0, 1\}$
- Objective: Maximize  $\sum_{i \in I} v_i x_i$
- Constraints:  $\sum_{i \in I} w_i x_i \leq C$

Maximize value of  
selected items

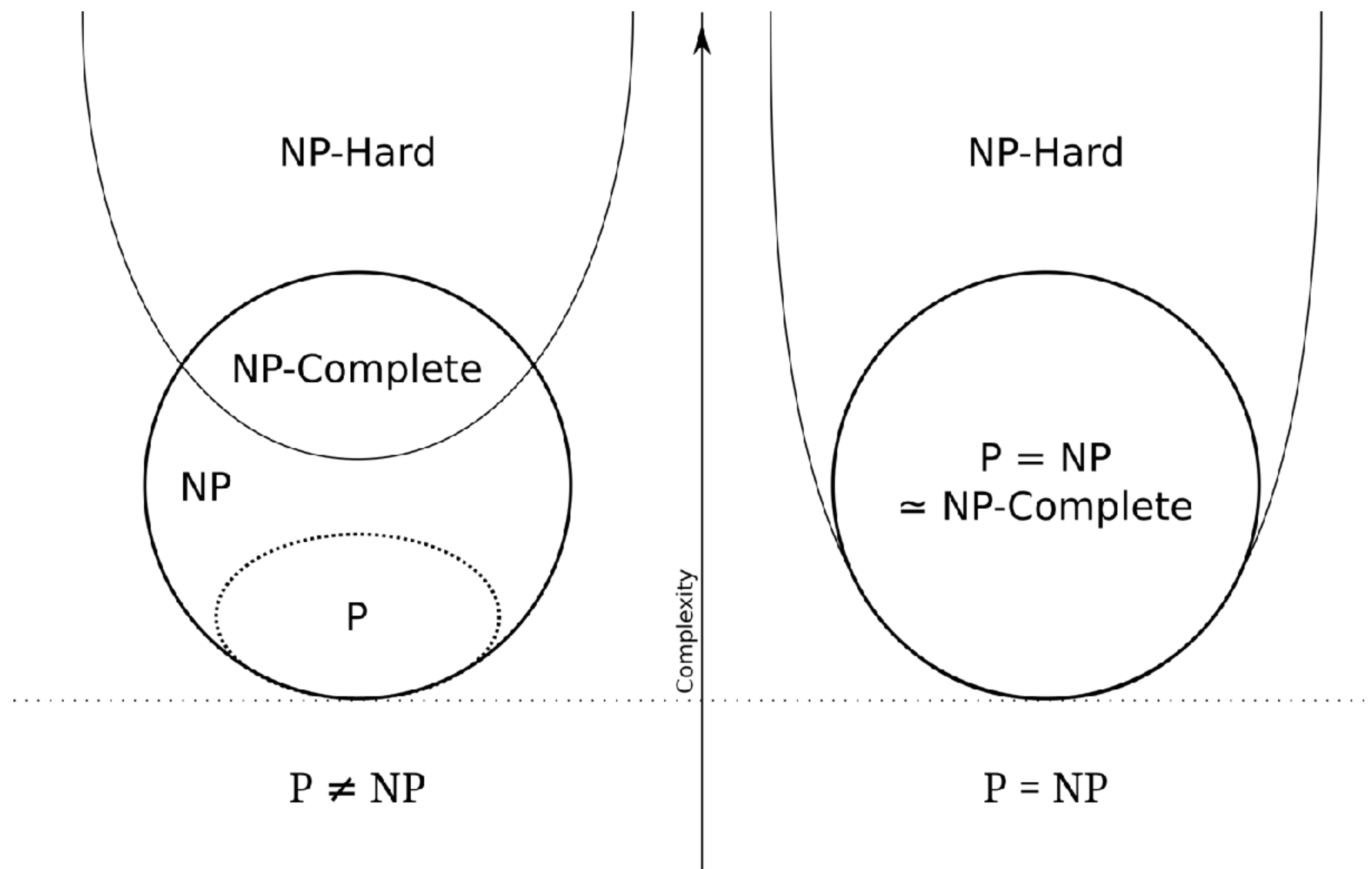
Under capacity  
constraint

# Reminder Calculability: P, NP and NP-Complete

- **P** is the class of problems with efficient algorithms for solving them (polynomial time in the size of the input)
- **NP** is the class of problems with efficient algorithm for (in)validating a candidate solution
- In NP, there are problems for which there no known algorithm for solving them efficiently (TSP for instance). We call those problems **NP-complete**. This is clique of problems because there is a know polynomial time reduction between any pairs of these.
- This is an open-question whether  $P = NP$  ?
- Nice read: Fortnow, L. (2009). The status of the P versus NP problem. *Communications of the ACM*, 52(9), 78-86.



# Summary



# Reminder Reduction

Suppose there is a polynomial-time reduction from problem A to problem B.

Is this statement true ?

- If A has no polynomial-time algorithm then neither does B

# Reminder Reduction

Suppose there is a polynomial-time reduction from problem A to problem B.

Is this statement true ?

- If A has no polynomial-time algorithm then neither does B 
- *Suppose that B would have a polynomial time algorithm, then it means that you could also solve A efficiently (using the reduced problem), which contradicts the hypothesis.*
- 
- A is the problem of finding a Hamiltonian Cycle in a graph (a **path** in a directed graph that visits each **vertex** exactly once)
- B is problem of sorting numbers.
-



# Reminder Reduction

Suppose there is a polynomial-time reduction from problem A to problem B.

Is this statement true ?

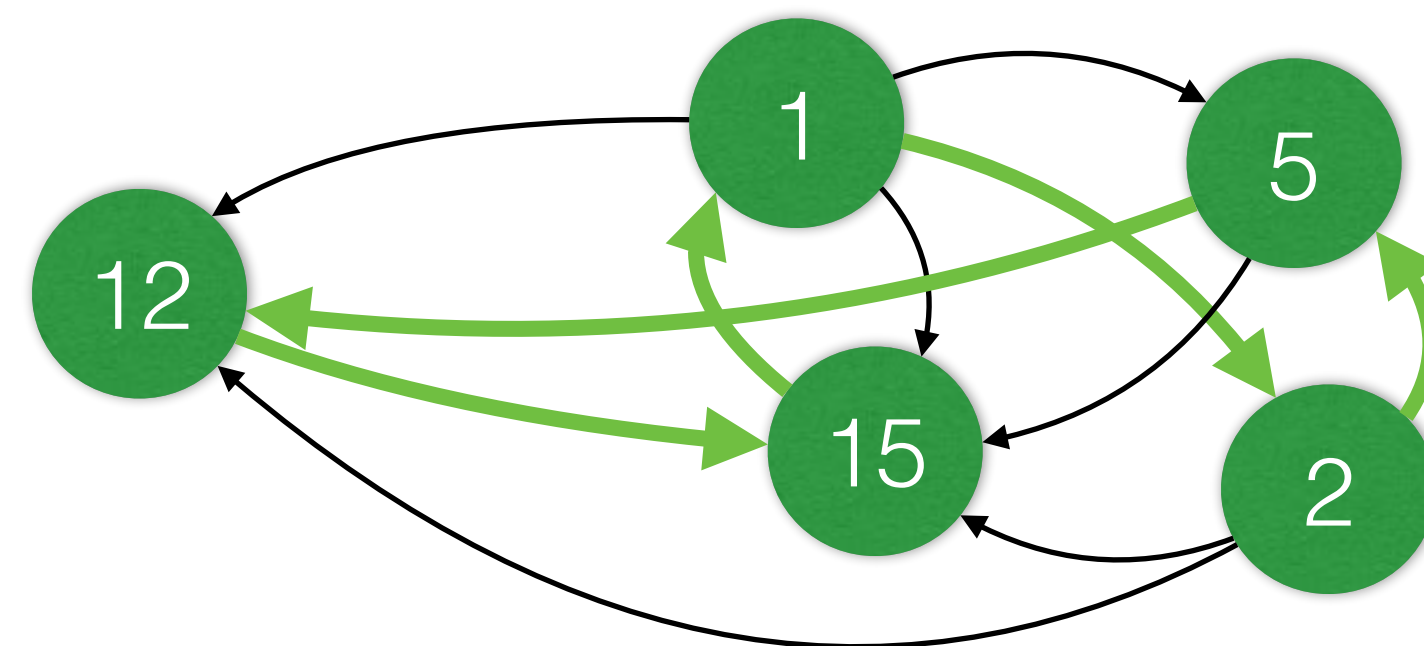
- If B has no polynomial-time algorithm then neither does A

# Reminder Reduction

Suppose there is a polynomial-time reduction from problem A to problem B.

Is this statement true ?

- If B has no polynomial-time algorithm then neither does A ❌
- Counter example:
  - A is problem of sorting numbers (efficient algorithms exist for sorting)
  - B is the problem of finding a Hamiltonian Cycle in a graph (a **path** in a directed graph that visits each **vertex** exactly once), known to be NP-Complete
  - Reduction: add an arc from  $i$  to  $j$  if  $i < j$  and from the largest to the smallest, a Hamiltonian cycle in this graph corresponds to visiting numbers in increasing order





# Reminder Reduction

Suppose there is a polynomial-time reduction from problem A to problem B.

Is this statement true ?

- If A is NP-hard and B has a polynomial-time algorithm then  $P=NP$

# Reminder Calculability

Suppose there is a polynomial-time reduction from problem A to problem B.

Is this statement true ?

- If A is NP-hard and B has a polynomial-time algorithm then  $P=NP$  

# First Question (important): Is this problem NP-Hard?

- Yes if the related decision problem is NP-Complete

$$\sum_{i \in I} v_i x_i \geq V$$

- $$\sum_{i \in I} w_i x_i \leq C$$

$$x_i \in \{0, 1\}$$

Natural numbers  $c_1, \dots, c_n, K$ .

Find  $S \subseteq \{1, \dots, n\}$  s.t.  $\sum_{j \in S} c_j = K$



# First Question (important): Is this problem NP-Hard?

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$$\sum_{i \in I} v_i x_i \geq V$$

$$\sum_{i \in I} w_i x_i \leq C$$

$$x_i \in \{0, 1\}$$

- We know *subset sum* is NP-Complete (a quite similar problem)

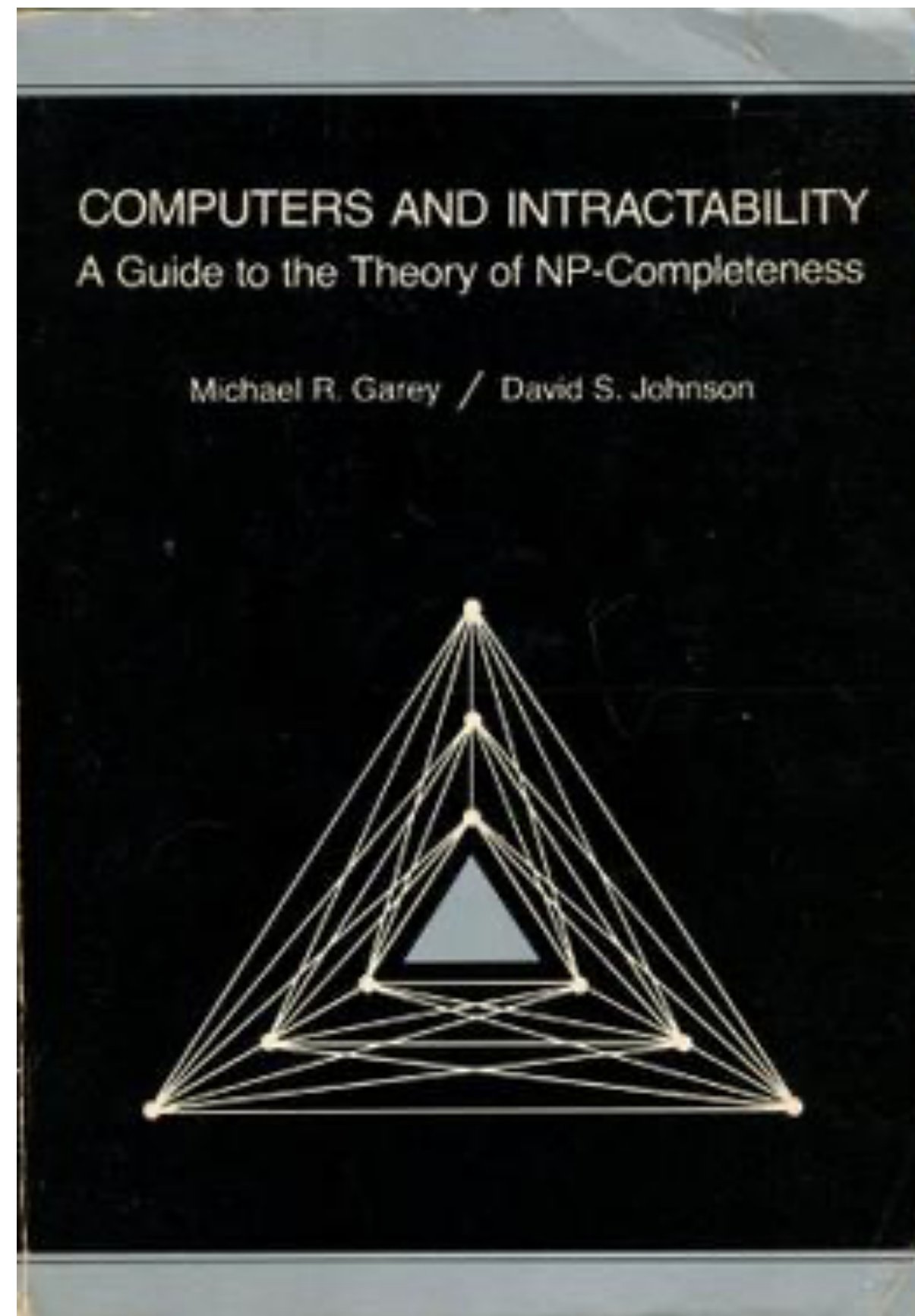
Natural numbers  $c_1, \dots, c_n, K$ .

Find  $S \subseteq \{1, \dots, n\}$  s.t.  $\sum_{j \in S} c_j = K$

- Exercise: Find a (polynomial) **reduction** from subset sum to knapsack (if you can solve knapsack efficiently, then you can solve subset sum efficiently ).

# A data-base of NP-Complete problems

This book is used a lot to make reductions for new problems:



This website is also useful <https://www.csc.kth.se/tcs/compendium/>

# A first bad news

- Most of the real-life constrained optimization problems are NP-hard.
- Does-it mean that we should run away and not solve them ?



- Of course not (at least I hope you will be convinced about that after having followed this course)

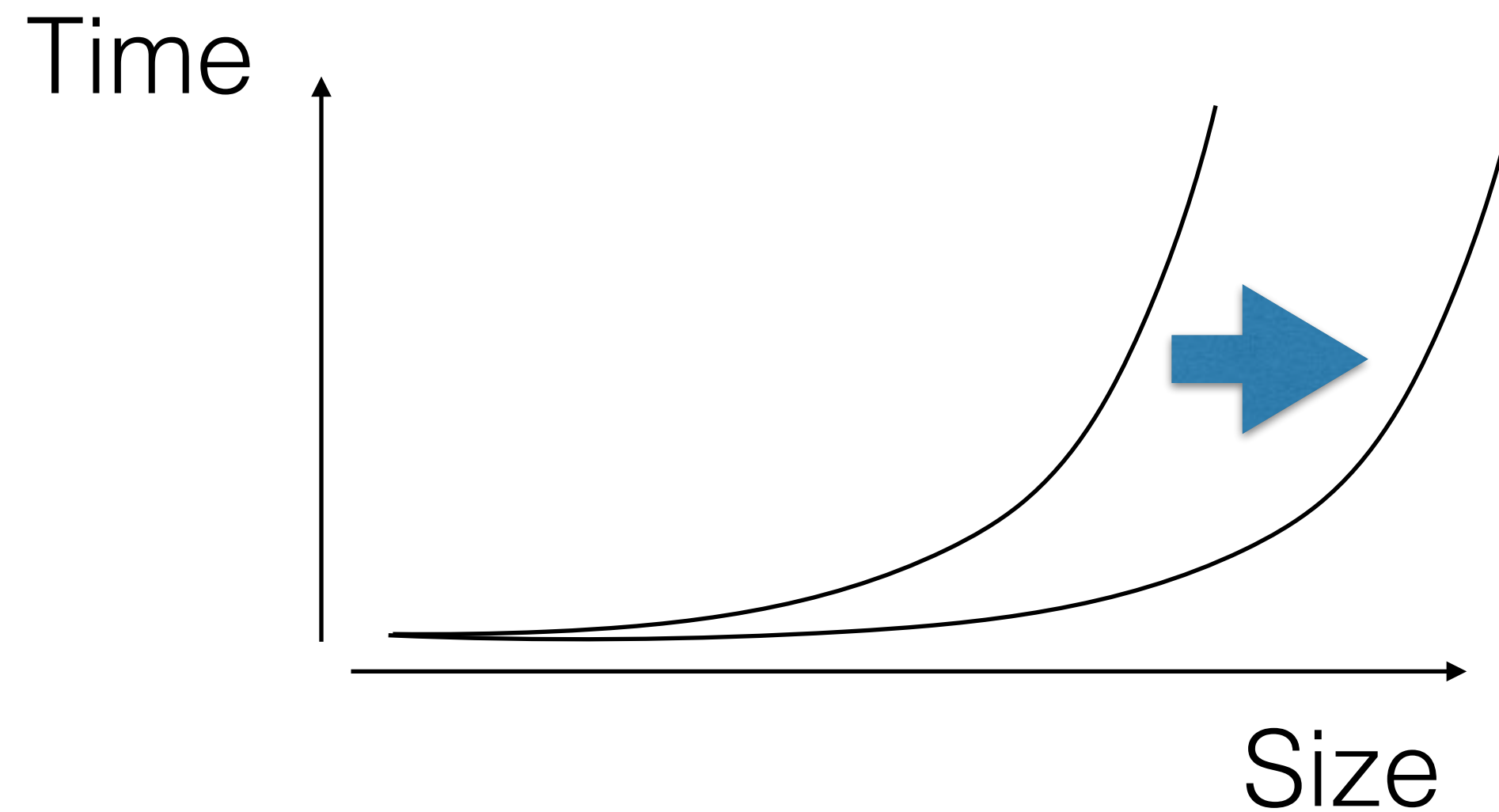


## Second question: what size are the instances ?

- Very small : A naive approach (like brute force) can easily find the best possible solution
- Try every possible solutions
  - $n$  items,  $2^n$  solutions
  - $n = 50$ , 1 ms to test one solution,  $> 30.000$  years

## Second question: what size are the instances ?

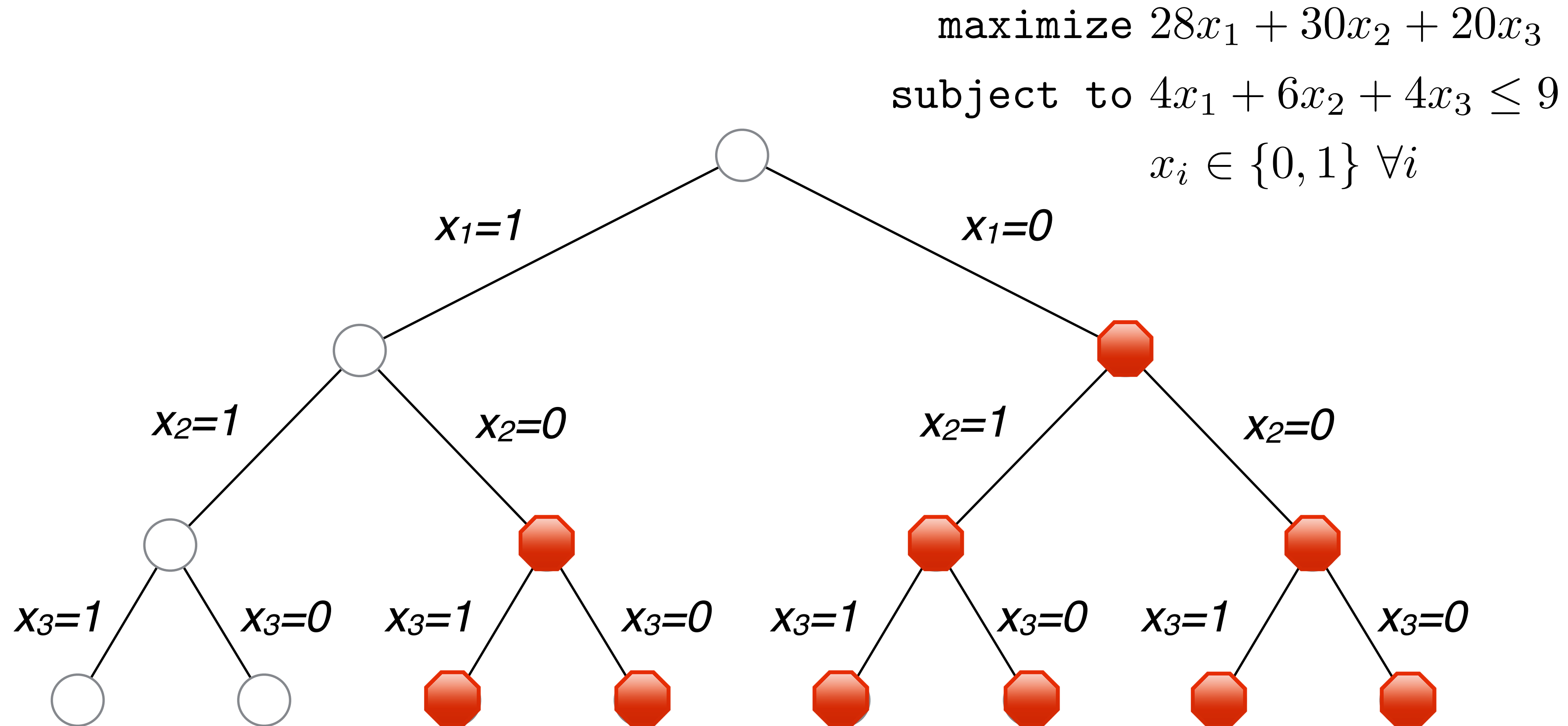
- Medium: I want to find the best solution



- Objective: shift the exponential behavior as much as possible to the right

# How to shift

- Almost all the exact approaches explore a search tree



- The goal is then to **prune this search tree** as much as possible to spare time (lower-bounding, dominance, etc)

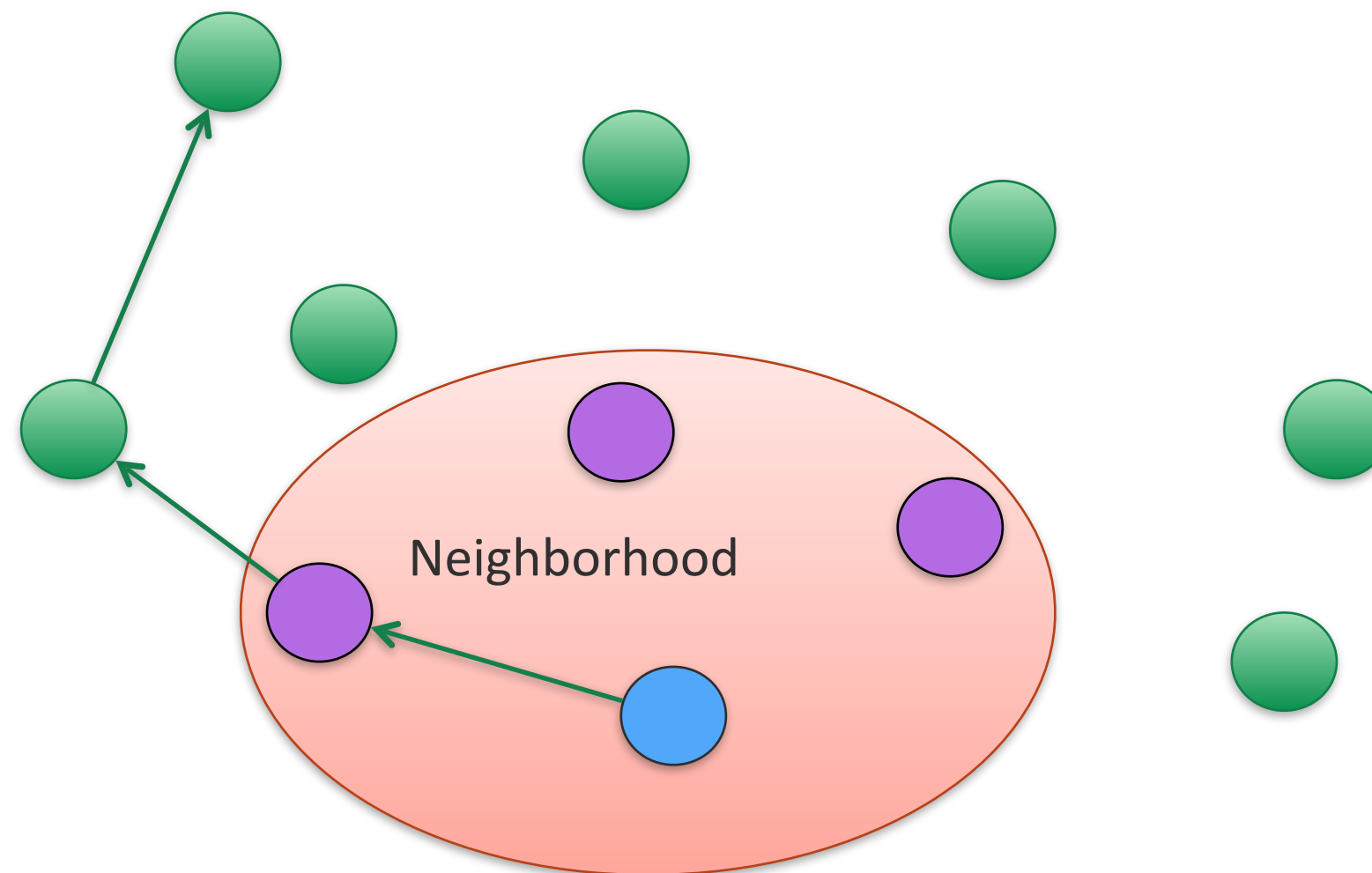


# The role of relaxation

- I have a solution to my knapsack (maximisation) problem with objective  $O$
- How far can it be from the optimal solution  $O^*$  ?
- Impossible to know, unless we have an upper bound that can be computed easily  $UB$ .
- Being pessimistic, I can guarantee that I am at most  $\frac{(O - UB)}{UB}$  suboptimal
- Being able to **find quickly good lower and upper-bounding** is **THE KEY INGREDIENT** in optimization
- Many techniques can help us: Linear Programming, Lagrangian relaxation, Dedicated Bounding algorithms, etc

## Second question: what size are the instances ?

- Medium: but you prefer to find a good enough solution quickly (within some time-limit)
- Local Search methods are generally the one we prefer in this case. Gradually apply small (improving) modifications to your solution. You can stop at any time. You can put a lot of intelligence in the exploration and the moves (as we will see in a few lectures)



## Second question: what size are the instances ?

- Large: Forget about proving optimality. You can generally only apply a greedy heuristic and a few local search moves depending on the time you have.
- Example: Knapsack, what would you do as a greedy heuristic ?

$$\begin{aligned} & \text{maximize} \sum_{i \in I} v_i x_i \\ & \text{subject to} \sum_{i \in I} w_i x_i \leq C \\ & \quad x_i \in \{0, 1\} \end{aligned}$$



# Other consideration

- What is the money budget for solving the problem
  - Do you have money to buy 10K \$ licence of a mixed integer programming solver ?
  - How much time do you have to develop a solution for your problem (usually also a cost budget that depends on the return).
- Funny (but important) fact: Your (future) salary may cost (much) more than a licence price. Developing a solution using an existing optimization solver might be the best solution for your employer. Commercial solvers are very good today (Gurobi, Cplex, CP-Optimizer, etc). Some company have an open-source model like Cosling

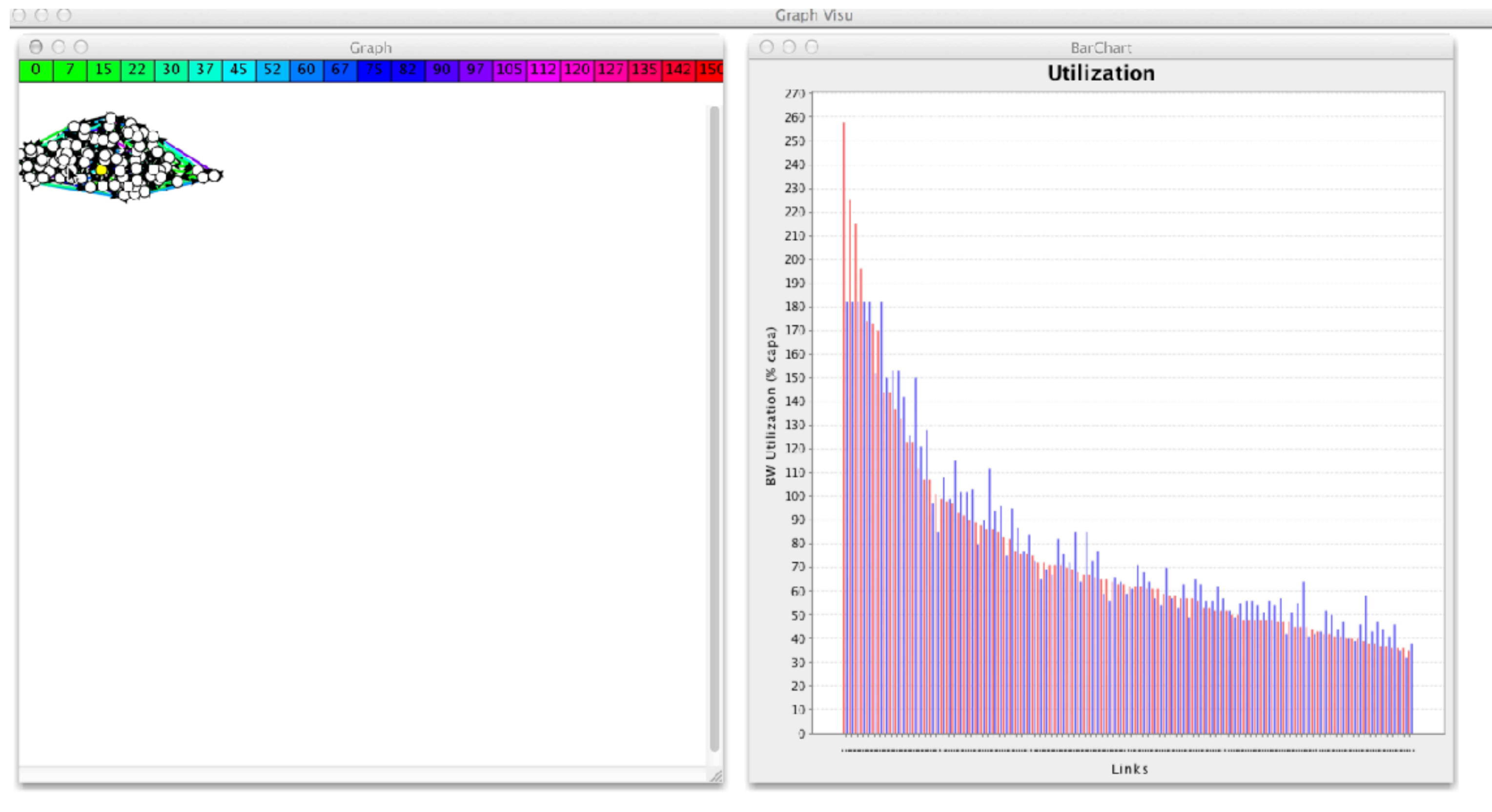
# Some Optimization Projects

- We are working on at the INGI department with my research team

# Computer Networks

- Traffic demand between every pair of nodes in a network (steady state, in MB per second).
- Traffic follow the shortest-path, you remember ?
- Two Traffic Engineering problems:
  - Find weights to put on each link to avoid congestion (accumulated traffic > capacity of the link).
  - Modify the paths (just a little, by introducing waypoints deviation) to avoid congestion

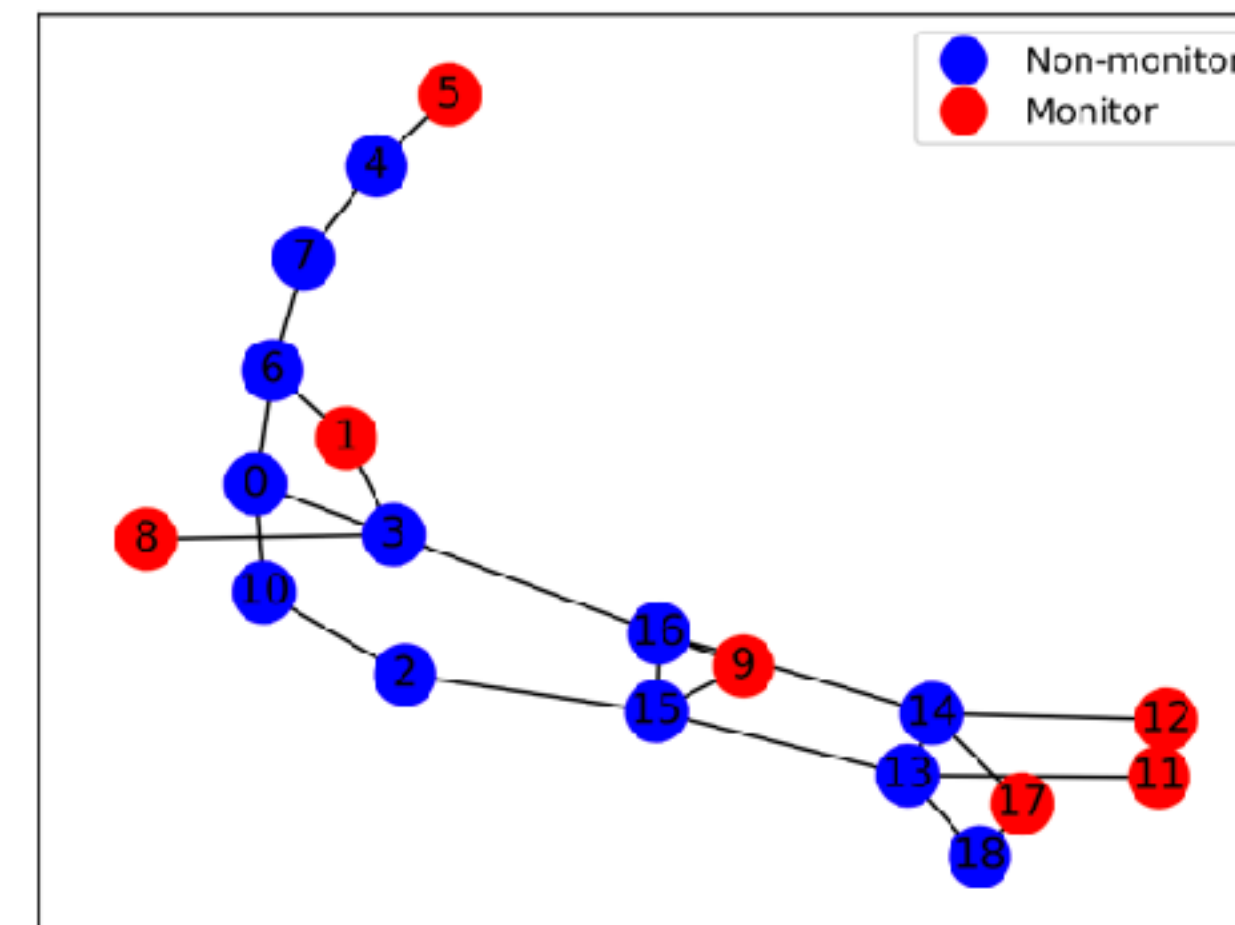




# Computer Network Monitoring

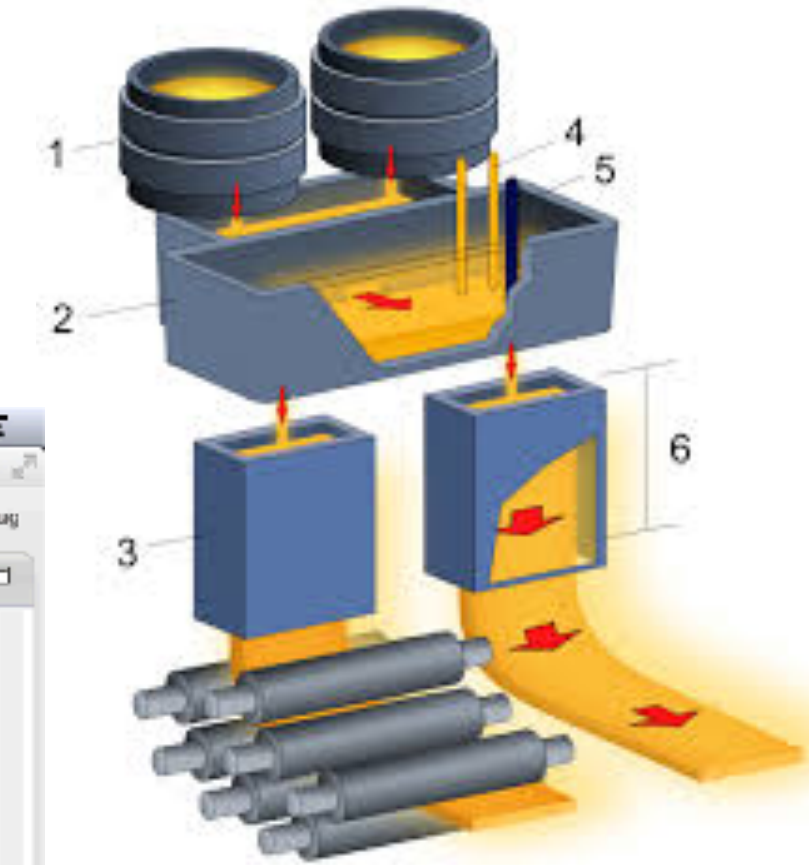
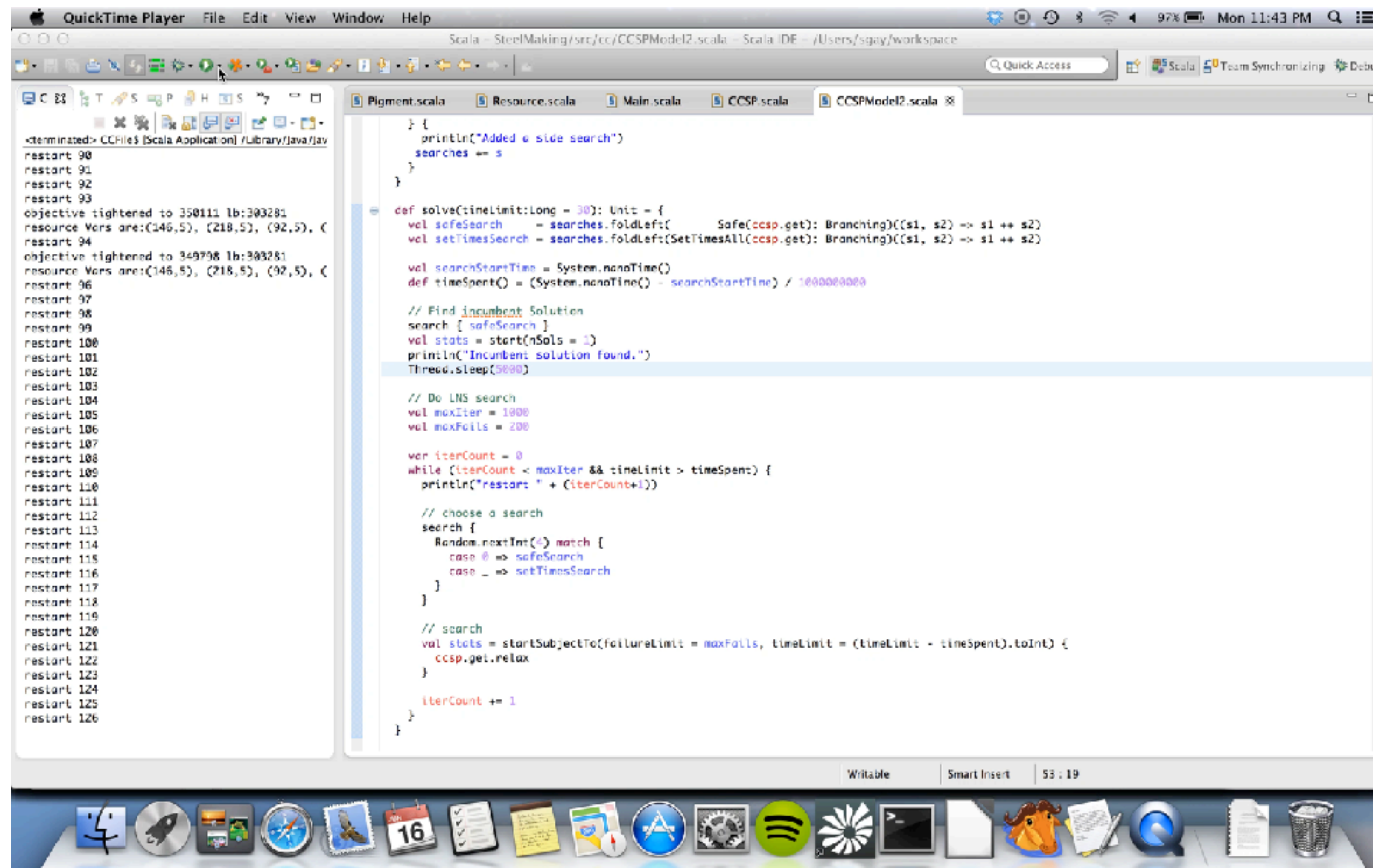
- Bad things can happen open a computer network.
- We want to detect it as soon as possible.
- Idea: introduce monitors exchanging between them. Whenever a path is broken, we know something is broken.
- Problem: Where do we place them so that there is a minimal number of them

A. Burlat, C. Pelsser, P. Schaus (2024). Boolean Network Tomography: CP and ILP Models for Effective Failure Detection and Diagnosis}. CPAIOR 2024





# Steel Production Planning



Gay, S., Schaus, P., & De Smedt, V. (2014). Continuous casting scheduling with constraint programming. In *Principles and Practice of Constraint Programming: 20th International Conference, CP 2014, L. Proceedings 20* (pp. 831-845). Springer International Publishing



# Rostering of a hospitality management school

- Create the roster for groups of students over the semester. For each group, each week (AM/PM), we must decide the course (atelier) to schedule

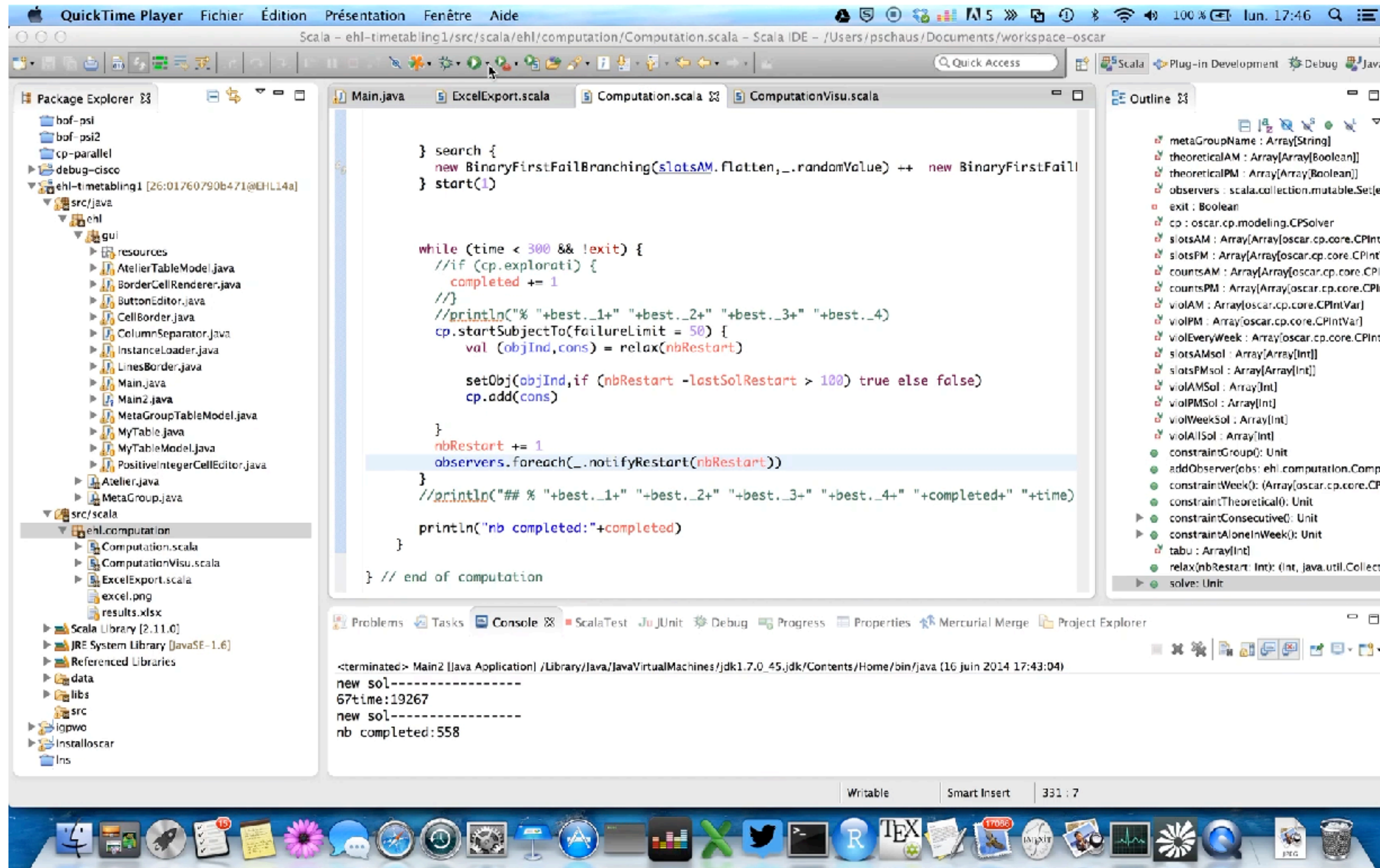
## Horizontal constraints:

- A theoretical course imposed in some slots
- For some courses: if scheduled must be scheduled two consecutive full weeks
- Some courses must be scheduled the whole week (morning and afternoon)
- Some courses can be scheduled a maximum number of times
- Some courses must be scheduled a minimum number of times (some equivalence)

## Vertical Constraints:

- Each course must be scheduled a min/max number of times each AM/PM
- Some courses must be scheduled every week
- Synchronize groups with a same mother language to give them theoretical courses at the same time

# Rostering of a hospitality management school

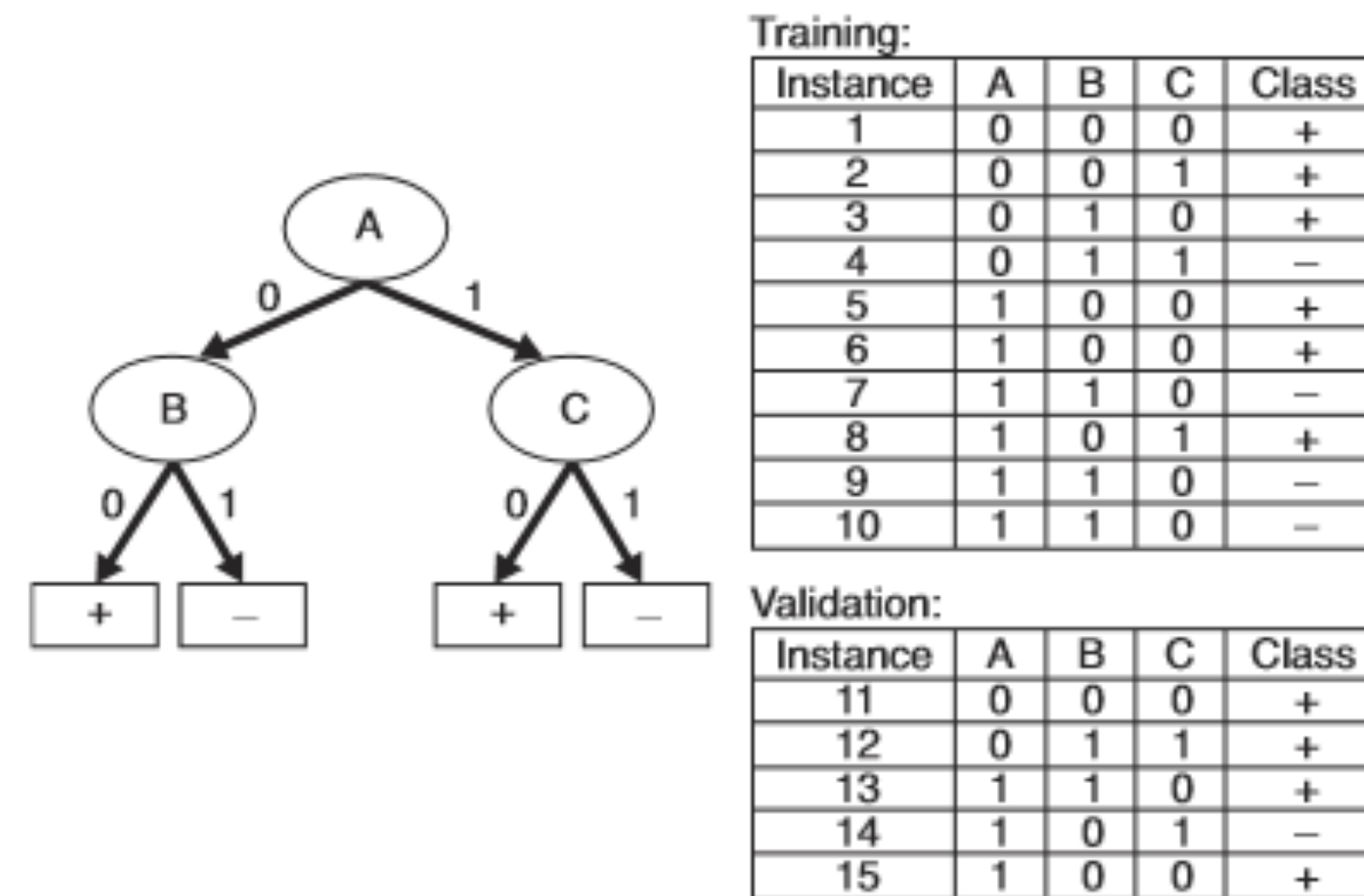


Schaus, P. (2013, November). Variable objective large neighborhood search: A practical approach to solve over-constrained problems. In *2013 IEEE 25th International Conference on Tools with Artificial Intelligence*



# Machine Learning: Decision Trees

- Find the decision tree (with limited depth) that minimizes the error rate on a training set

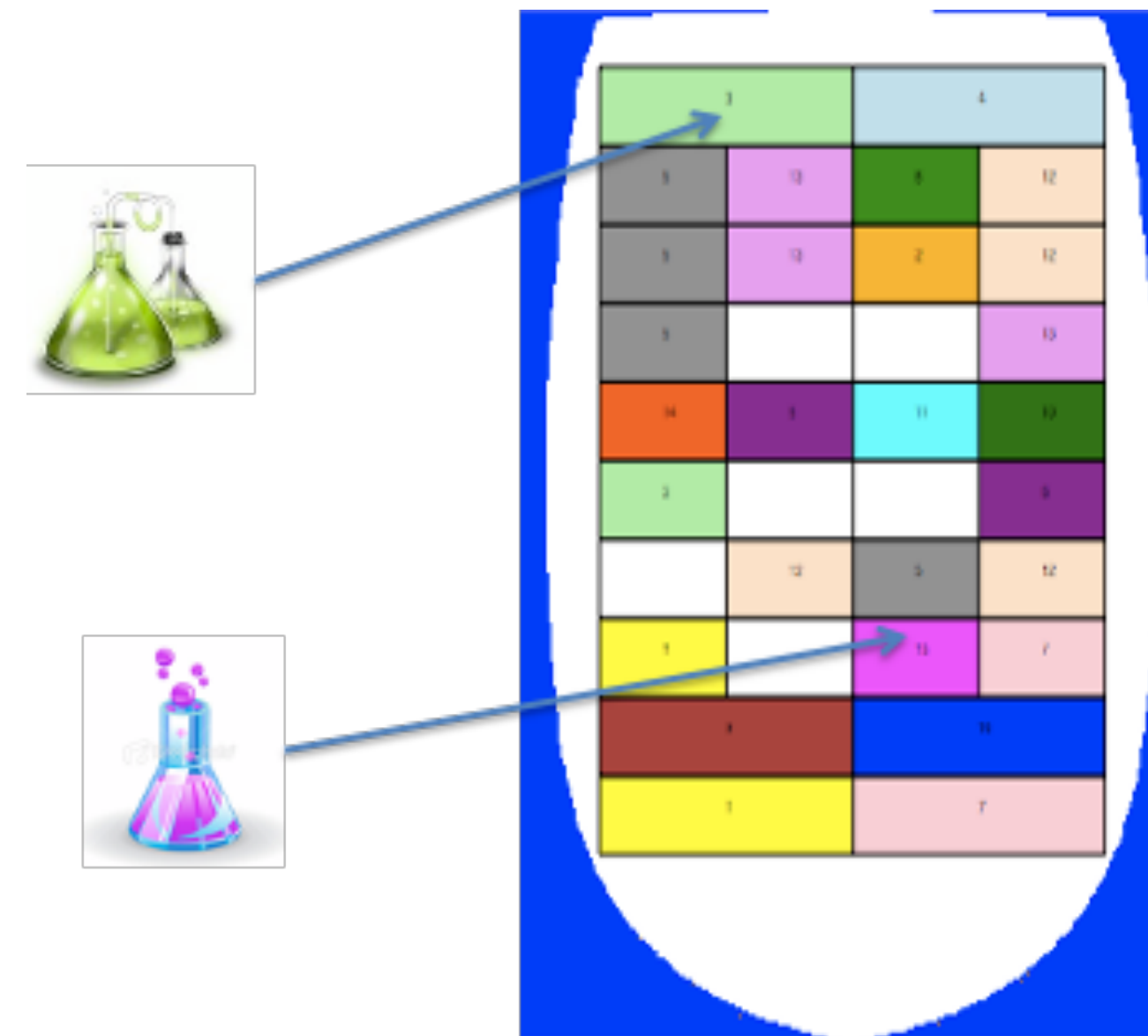


<https://github.com/aia-uclouvain/pydl8.5>

Aglin, G., Nijssen, S., & Schaus, P. (2020, April). Learning optimal decision trees using caching branch-and-bound search. In *Proceedings of the AAAI conference on artificial intelligence* (Vol. 34, No. 04, pp. 3146-3153).

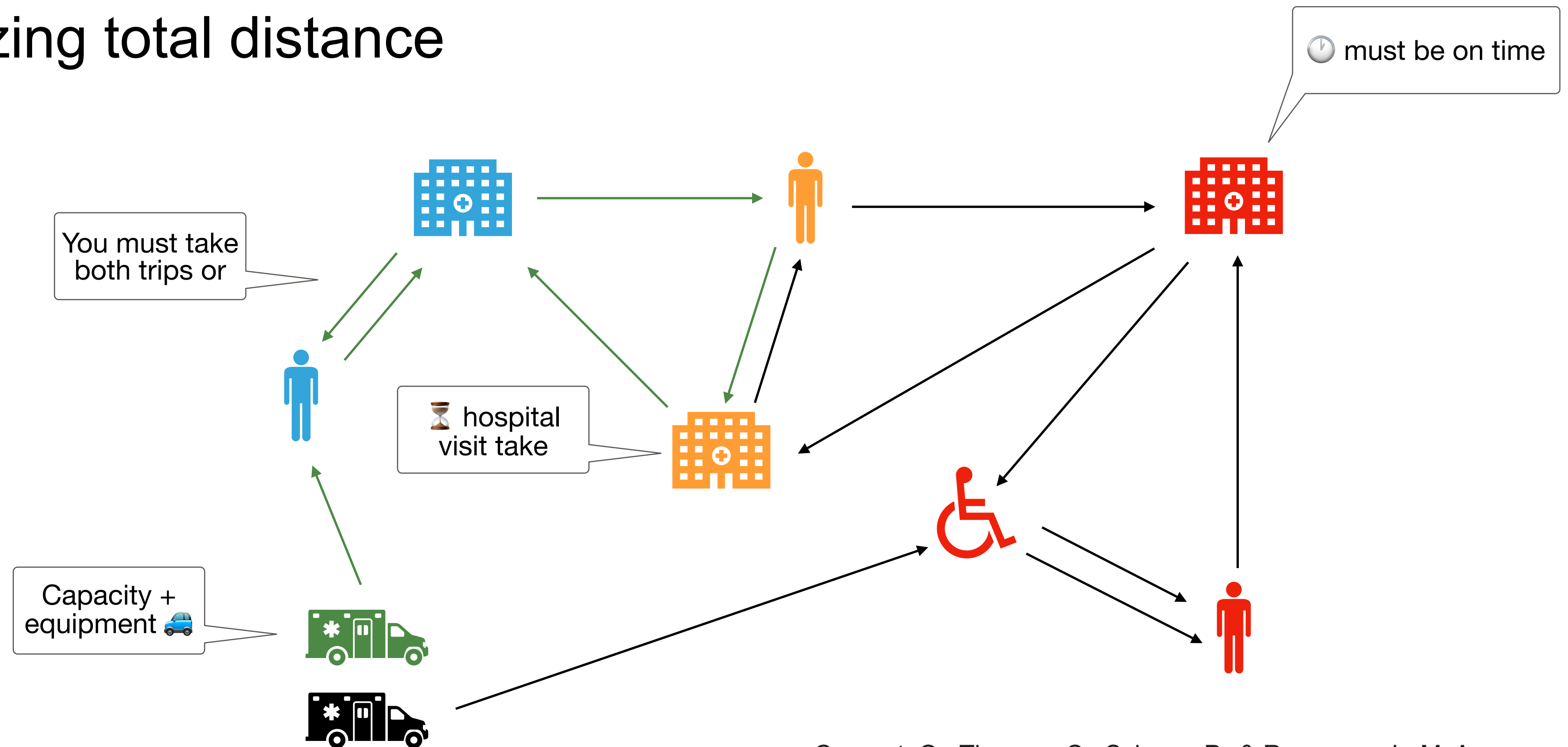
# Chemical Tanker

- For each product an amount (liters) to place is given
  - Each tank has a maximum capacity (in liters)
  - A set of chemical products must be placed into tanks on a tanker
- Objective:
- Maximize the empty/unused space while placing all the products



# Patient Transportation Problem (PTP)

- 1: maximize the number of served requests
- 2: minimizing total distance

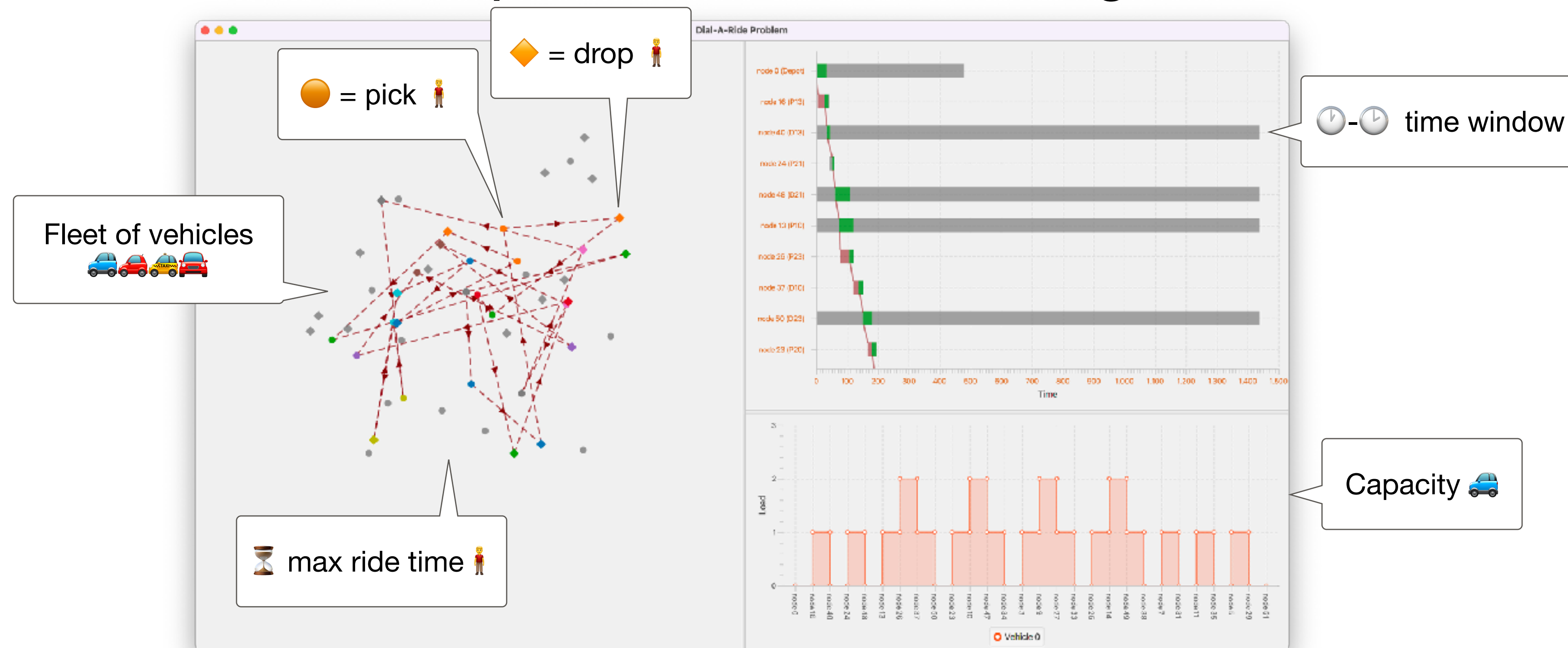


Cappart, Q., Thomas, C., Schaus, P., & Rousseau, L. M. A  
constraint programming approach for solving patient transportation  
problems. CP2018



# Dial A Ride Problem

- Serve all the requests while minimizing total distance





# Air Plane Dismantling

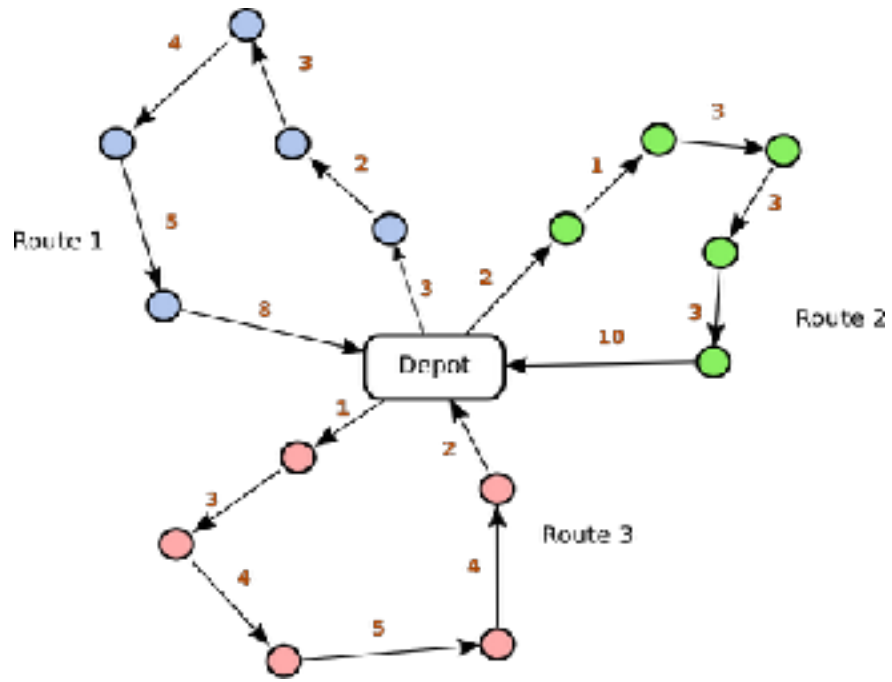
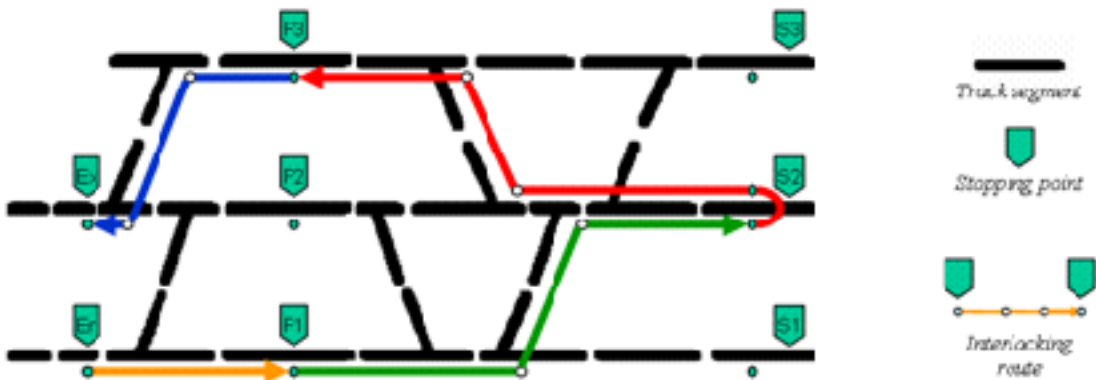
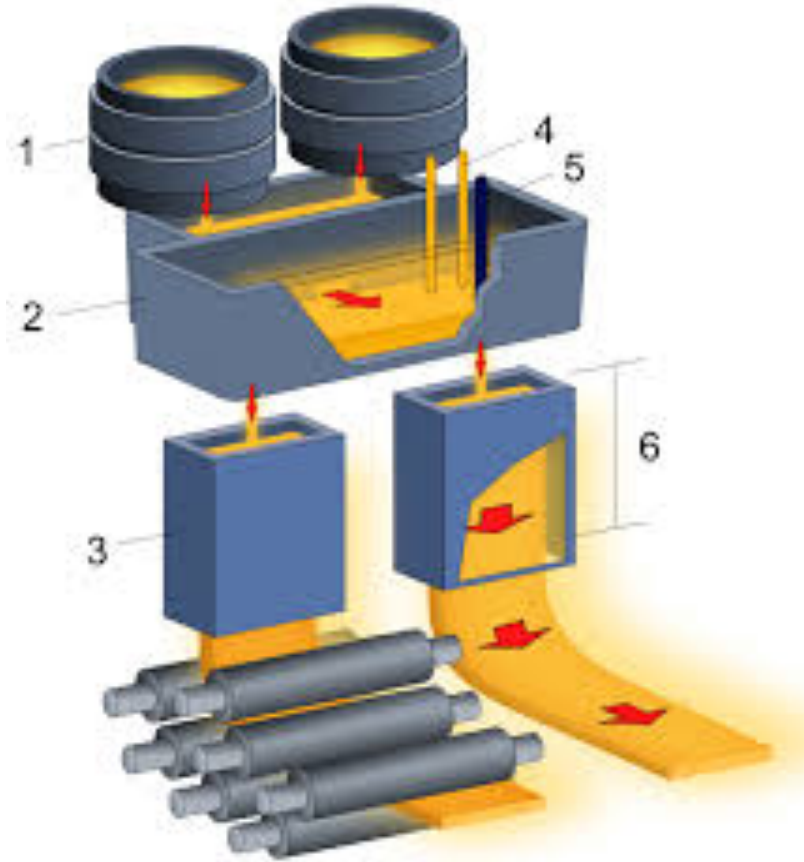
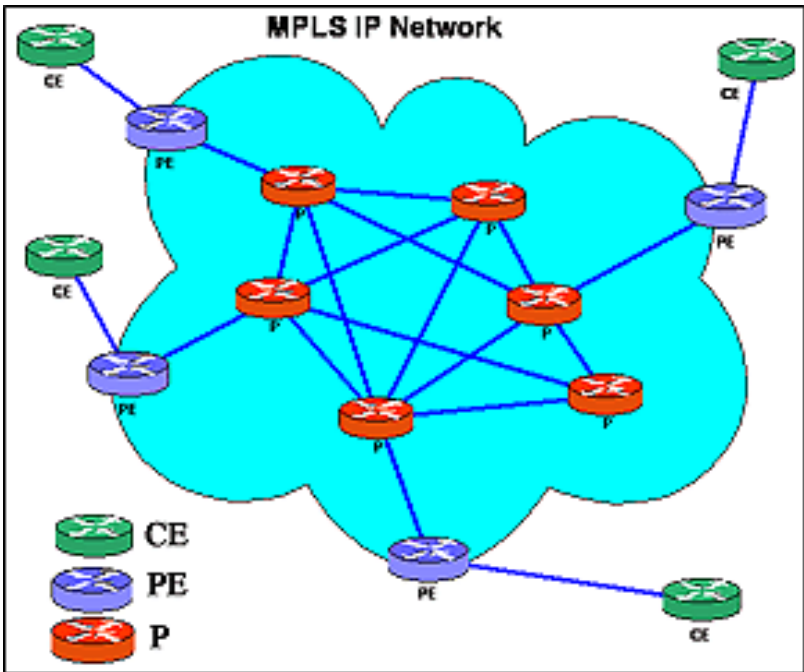
- Schedule the dismantling of a plane (with Charles Thomas)
  - Worker skills constraints
  - Precedence constraints
  - Tools and equipment constraints
  - Value objective, minimize makespan



Thomas, C., & Schaus, P. (2024, May). A Constraint Programming Approach for Aircraft Disassembly Scheduling. In *International Conference on the Integration of Constraint Programming, Artificial Intelligence, and Operations Research* (pp. 211-220). Cham: Springer Nature Switzerland.



# All the techniques we will see are used!



# Techniques

- Dynamic Programming
- Branch and Bound
- Lagrangian Relaxation
- Linear Programming
- Network Flows
- Constraint Programming (course LINFO2365 dedicated to this topic in Q2)
- Local Search
- Decision Diagrams

# Advanced Algorithms for Optimization

LINFO2266

## Introduction

Prof: Pierre Schaus  
TAs: Alice Burlats, Amaury Guichard

